SCEC Code Verification: A Proposed 3D Benchmark of Thermal Pressurization and Rate-and-State Strong Velocity Weakening Friction

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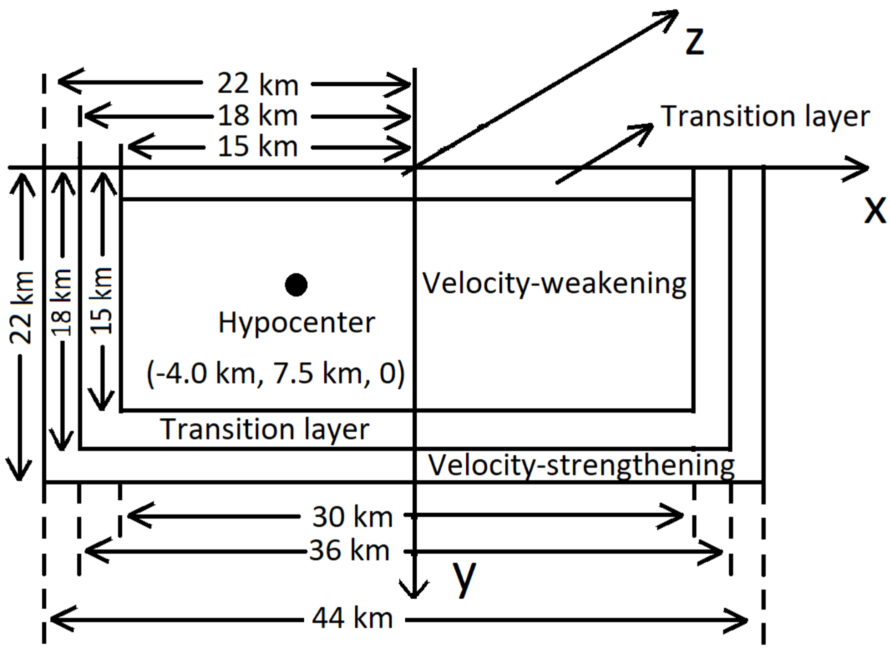
1 Model Geometry

A planar fault lies in an isotropic, linear elastic whole-space. The medium is homogeneous having constant density ($\rho = 2670\,\mathrm{kg/m^3}$), S-wave speed ($c_s = 3.464\,\mathrm{km\,s^{-1}}$), and P-wave speed ($c_p = 6\,\mathrm{km\,s^{-1}}$). To simplify later expressions, a coordinate system will be adopted in which the fault is the plane z = 0, with the hypocenter located at (x_0, y_0) = (-4.0,7.5 km). This is shown in the figure below. Note the new hypocenter location relative to that used in other TPV benchmark problems. The central portion of the fault, $-W < x < W$, $w < y < W$, with $W = 15$ km and w=3 km, is velocity-weakening. A transition layer of width $w = 3$ km in which the frictional properties continuously change from velocity-weakening to velocity-strengthening surrounds the central velocity-weakening region of the fault. Outside of the transition region, the fault is velocity-strengthening up to 4 km.



2 Friction Law

Let $\boldsymbol{\tau} = (\tau_x,\tau_y)$ be the shear traction vector (specifically, the traction exerted by the positive side of the fault on the negative side), the magnitude of which is $\tau = \sqrt{\tau_x^2 + \tau_y^2}$, and let σ be the total normal stress acting on the fault, taken to be positive in compression. In terms of the components of the total stress tensor σ_{ij}, $\tau_x = \sigma_{zx}$, $\tau_y = \sigma_{zy}$, and $\sigma = -\sigma_{zz}$. The fault zone is assumed to be fluid-saturated with pore pressure p. The pore pressure may vary both along the fault and perpendicular to it: $p = p(x, y, z)$; the pore pressure on the fault is $p_f = p_f(x, y) = p(x, y, 0)$. The effective normal stress (again, positive in compression) is $\overline{\sigma} = \sigma - p_f$. Let $\boldsymbol{V} = (V_x, V_y)$ be the slip velocity vector, the magnitude of which is $V = \sqrt{V_x^2 + V_y^2}$, and let $\boldsymbol{\delta} =$

(δ_x, δ_y) be the slip vector. Slip is defined as the displacement discontinuity across the fault: $\delta_i = u_i(x, y, 0^+) - u_i(x, y, 0^-)(i = x, y)$ where $u_i(x, y, z)$ is the displacement field. Likewise, $V_i = v_i(x, y, 0^+) - v_i(x, y, 0^-)(i = x, y)$ where $v_i(x, y, z)$ is the particle velocity. Finally, let Ψ be the state variable on the fault. The shear traction is always equal to the shear strength of the fault, which is the product of the friction coefficient and effective normal stress:

$$\tau = f(V, \Psi)\overline{\sigma} \tag{1}$$

The friction law is the same as TPV 103/104. The friction coefficient is a function of V and Ψ.

$$f(V, \Psi) = a \ \sinh^{-1}\left[\frac{V}{2V_0}\, exp\left(\frac{\Psi}{a}\right)\right] \tag{2}$$

The state variable evolves according to the equation

$$\frac{d\Psi}{dt} = -\frac{V}{L}[\Psi - \Psi_{\mathrm{SS}}(V)], \tag{3}$$

$$\Psi_{\mathrm{SS}}(V) = a\, ln\left\{\frac{2V_0}{V}\sinh\left[\frac{f_{\mathrm{SS}}(V)}{a}\right]\right\}. \tag{4}$$

$f_{\mathrm{SS}}(V)$ is the steady state friction coefficient, which depends on V and the friction law parameters f_0, V_0, a, b, f_{W}, and V_{W}:

$$f_{\mathrm{SS}}(V) = f_{\mathrm{W}} + \frac{f_{\mathrm{LV}}(V) - f_{\mathrm{W}}}{[1 + (V/V_{\mathrm{W}})^8]^{1/8}}, \tag{5}$$

with a low-velocity steady state friction coefficient

$$f_{\mathrm{LV}}(V) = f_0 - (b - a)\,\ln(V/V_0). \tag{6}$$

The slip velocity vector points in the direction of the shear traction vector:

$$\boldsymbol{\tau}/\tau = \boldsymbol{V}/V \tag{7}$$

The friction law parameters are given in the table below. Note that with the exception of a and V_{W}, they are uniform on the fault.

f_0	V_0	$a(x,y)$	b	L	f_{W}	$V_{\mathrm{W}}(x,y)$
0.6	$1 \times 10^{-6}\,\mathrm{m\,s^{-1}}$	$0.01 + \Delta a(x,y)$	0.014	$0.4\,\mathrm{m}$	0.2	$0.1\,\mathrm{m\,s^{-1}} + \Delta V_{\mathrm{W}}(x,y)$

To stop the rupture, the friction law changes from velocity-weakening in the rectangular interior region of the fault to velocity-strengthening sufficiently far outside this region. The transition occurs smoothly within a transition layer of width $w = 3$ km. Outside the transition layer, the fault is made velocity-strengthening by increasing a by $\Delta a_0 = 0.01$ and V_{W} by $\Delta V_{\mathrm{W}0} = 0.9$. The changes in a and V_{W}, which are added to the values of a and V_{W} in the velocity-weakening interior of the fault, are

$$\Delta a(x,y) = \Delta a_0[1 - B_1(x; W, w)\, B_2(y; W, w)] \tag{8}$$

$$\Delta V_w(x,y) = \Delta V_{\mathrm{W}0}[1 - B_1(x; W, w)\, B_2(y; W, w)], \tag{9}$$

in which

$$B_1(x;W,w) = \begin{cases} 1, \ |x| \leq W \\ 0.5\left[1+\tanh\left(\frac{w}{|x|-W-w}+\frac{w}{|x|-W}\right)\right], \ W < |x| < W+w \\ 0, \ |x| \geq W+w \end{cases} \tag{10}$$

$$B_2(y;W,w) = \begin{cases} 0.5\left[1+\tanh\left(\frac{w}{w-y}-\frac{w}{y}\right)\right], \ y < w \\ 1, \ w \leq y \leq W \\ 0.5\left[1+\tanh\left(\frac{w}{y-W-w}+\frac{w}{y-W}\right)\right], \ W < y < W+w \\ 0, \ y \geq W+w \end{cases} \tag{11}$$

B_i are mathematically smooth versions of the boxcar function (meaning that B_i and all of their derivatives are continuous).

3 Thermal Pressurization

In addition to changes in the friction coefficient, the fault strength can also be altered by changes in pore pressure on the fault in response to shear heating. Conservation of energy and fluid mass, together with Fourier's law and Darcy's law and several assumptions including neglecting advection, gives the following equations governing temperature T and pore pressure p

in the fault zone:

$$\frac{\partial T}{\partial t} = \alpha_{\text{th}} \frac{\partial^2 T}{\partial z^2} + \frac{\tau V}{\rho c h \sqrt{2\pi}} \exp\left(\frac{-z^2}{2h^2}\right), \tag{12}$$

$$\frac{\partial p}{\partial t} = \alpha_{\text{hy}} \frac{\partial^2 p}{\partial z^2} + \Lambda \frac{\partial T}{\partial t}, \tag{13}$$

in which α_{th} is the thermal diffusivity, α_{hy} is the hydraulic diffusivity, ρc is the volumetric heat capacity, and Λ quantifies the undrained thermal pressurization response (i.e., the pore pressure increase per unit increase in temperature). We have assumed a finite width shear zone in which the shear strain rate distribution has a Gaussian shape with width h. Equations 12 and 13 hold at each point (x, y) on the fault. Both boundary and initial conditions are required for T and p. The initial conditions are the constant values $T_{\text{ini}} = 483.15$ K and $p_{\text{ini}} = 80$ MPa. Under the assumptions of spatially uniform properties in the z-direction and a localized region of shear heating, appropriate boundary conditions are $T \to T_{\text{ini}}$ and $p \to p_{\text{ini}}$ as $z \to \pm\infty$. Due to the symmetry of T and p about $z = 0$, the problem can be reduced to one for only $z \geq 0$ with the boundary conditions $\partial T/\partial z = \partial p/\partial z = 0$ at $z = 0$. The thermal pressurization parameters are given in the table below. They are all uniform on the fault.

α_{th}	ρc	Λ	h
$1 \times 10^{-6}\,\text{m}^2/\text{s}$	$2.7\,\text{MJ}/\text{m}^3\text{K}$	$0.1\,\text{MPa}\,\text{K}^{-1}$	$20\,\text{mm}$

$$\alpha_{\text{hy}}(x, y) = 4.0 \times 10^{-4} + \Delta\alpha_{\text{hy}}(x, y) \tag{14}$$

The hydraulic diffusivity α_{hy} is uniform ($= 4 \times 10^{-4}\,\mathrm{m}^2/\mathrm{s}$) within velocity weakening portion of the fault. α_{hy} smoothly changes in the transition layer following the boxcar function in Eq. 10,

$$\Delta\alpha_{\text{hy}}(x, y) = \Delta\alpha_{hy0}\,[1 - B_1(x; W, w)B_3(y; W, w)], \tag{15}$$

where, $\Delta\alpha_{hy0} = 1\,\mathrm{m}^2/\mathrm{s}$ and

$$B_3(y; W, w) = \begin{cases} 1, \;\; y \leq W \\ 0.5\left[1 + \tanh\left(\frac{w}{y-W-w} + \frac{w}{y-W}\right)\right], \;\; W < y < W + w \\ 0, \;\; y \geq W + w. \end{cases} \tag{16}$$

4 Initial Conditions

At t=0, the fault is everywhere sliding in the horizontal direction with initial velocity $V = V_{\text{ini}}$ ($= 1 \times 10^{-16}\,\mathrm{m\,s^{-1}}$). The initial shear stress on the fault, which is also horizontal, is $\tau_{\text{ini}}(x, y)$, the effective normal stress is $\overline{\sigma}_{\text{ini}}(x, y)$, and the initial value of the state variable is $\Psi_{\text{ini}}(x, y)$.

The initial normal stress and shear stress (in Pa) are given by:

$$\overline{\sigma}_{\text{ini}}(x, y) = \min((\rho - \rho_w)gy,\, 45 \times 10^6) \tag{17}$$

$$\tau_{\text{ini}}(x, y) = 0.41 \times \overline{\sigma}_{\text{ini}}(x, y) \tag{18}$$

With $\rho_w = 1000$ kg/m^3 the water density and $g = 9.8$ m/s^2 the gravitational acceleration.

From equations 1 and 2, it follows that

$$\Psi_{\text{ini}}(x, y) = a \ln \left[\frac{2V_0}{V_{\text{ini}}} \sinh \left(\frac{\tau_{\text{ini}}}{a\overline{\sigma}_{\text{ini}}} \right) \right] \tag{19}$$

In the medium surrounding the fault, the only nonzero stresses are the horizontal shear stress and the total normal stress component acting on the fault; these values are uniform and identical to those on the fault:

$$\sigma_{zx}(x, y, z) = \tau_{\text{ini}} \; and \; \sigma_{zz}(x, y, z) = -\sigma_{\text{ini}} \; at \; t = 0. \tag{20}$$

The medium is initially moving with equal and opposite horizontal velocities of $V_{\text{ini}}/2$ on the two sides of the fault:

$$v_x(x, y, z) = \begin{cases} V_{\text{ini}}/2, \;\; z > 0 \\ -V_{\text{ini}}/2, \;\; z < 0 \end{cases} \quad at \; t = 0. \tag{21}$$

Displacement in the medium and slip on the fault are measured from zero at t = 0.

5 Nucleation Method

Starting at t = 0, rupture is nucleated by imposing a horizontal shear traction perturbation (i.e., a perturbation to τ_x) that depends on both space and time. The particular form is such that the perturbation smoothly grows from zero to its maximum amplitude $\Delta\tau_0$ over a finite time interval T (not to be confused with temperature), and is confined to a finite region of the fault of radius R. The perturbation is mathematically smooth in time and space (i.e., the function and all derivatives are continuous). Specifically, the perturbation is

$$\Delta\tau(x,y,t) = \Delta\tau_0 F\left(\sqrt{(x-x_0)^2+(y-y_0)^2}\right) G(t), \tag{22}$$

in which

$$F(r) = \begin{cases} \exp\left(\frac{r^2}{r^2-R^2}\right), & r < R \\ 0, & r \geq R \end{cases} \tag{23}$$

and

$$G(t) = \begin{cases} \exp\left(\frac{(t-T)^2}{t(t-2T)}\right), & 0 < t < T \\ 1, & t \geq T \end{cases} \tag{24}$$

The perturbation is radially symmetric, with the radial distance away from the hypocenter along the fault given by $r = \sqrt{(x-x_0)^2+(y-y_0)^2}$. The nucleation parameters are given in the table below.

$\Delta\tau_0$	R	T	(x_0, y_0)
50 MPa	1.5 km	1 s	(−4 km, 7.5 km)

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The following data should be submitted to the code validation website (where instructions for the data file formats can be found):

Time Histories of Fields at Fault Stations:

Report the complete time histories from t = 0 to t = 15 s of both components of slip (δ_x and δ_y) and slip velocity (V_x and V_y), both components of shear traction (τ_x and τ_y), the effective normal stress $\overline{\sigma}$, the state variable (Ψ), and temperature and pressure on the fault (T and p) at thirteen stations on the fault:

x (km)	0	0	0	9	12	12	15	18	-9	-12	-12	-15	-18
y (km)	3	7.5	12	7.5	3	12	7.5	7.5	7.5	3	12	7.5	7.5

Rupture Front Arrival Times:

Report the rupture front arrival time at all points within the fault (-22 km $< x <$ 22 km, 0 km $< y <$ 22 km). The rupture front arrival time is defined as the time at which the slip velocity, V, first exceeds 1 mm/s.

Time Histories of Fields at Free Surface Stations:

Report the complete time histories from t = 0 to t =15 s of all components of displacement and particle velocity at each of the following six stations on the free surface:

x (km)	0	0	12	12	-12	-12
z (km)	9	-9	6	-6	6	-6