

TPV26 and TPV27

Vertical Fault with Viscoplasticity Benchmarks

January 9, 2014

These 3D benchmarks use a single planar vertical fault in a half-space. We are doing two benchmarks, as shown in the following table.

Benchmarks			
<i>Benchmark</i>	<i>Dimension</i>	<i>Rupture Type</i>	<i>Material Properties</i>
TPV26	3D	Right-lateral, vertical strike-slip.	Linear elastic.
TPV27	3D	Right-lateral, vertical strike-slip.	Drucker-Prager viscoplastic.

We request that you run each of these two benchmarks using two resolutions: 100 meter resolution, and 50 meter resolution.

Benchmark Summary

- The geometry is a single, vertical, planar, strike-slip fault in a half-space. The fault is 40 km long and 20 km deep.
- Material properties are uniform throughout the model volume. The benchmarks include gravity and fluid pressure. The initial stress tensor varies with depth, and is in static equilibrium.
- The material properties are the only difference between the two benchmarks.
- The benchmarks use linear slip-weakening friction. Friction parameters are uniform over the fault surface, except that there is increased frictional cohesion in the uppermost 5 km to suppress free-surface effects.
- The fault boundary condition is that slip goes to zero at the border of the fault. So, a node which lies precisely on the border of a fault should *not* be permitted to slip. The free surface is not considered to be a border of the fault.
- Nucleation is done using a smoothed forced rupture. The forced rupture is applied over an interval of 0.5 seconds, and the forced rupture velocity decreases with distance from the hypocenter.

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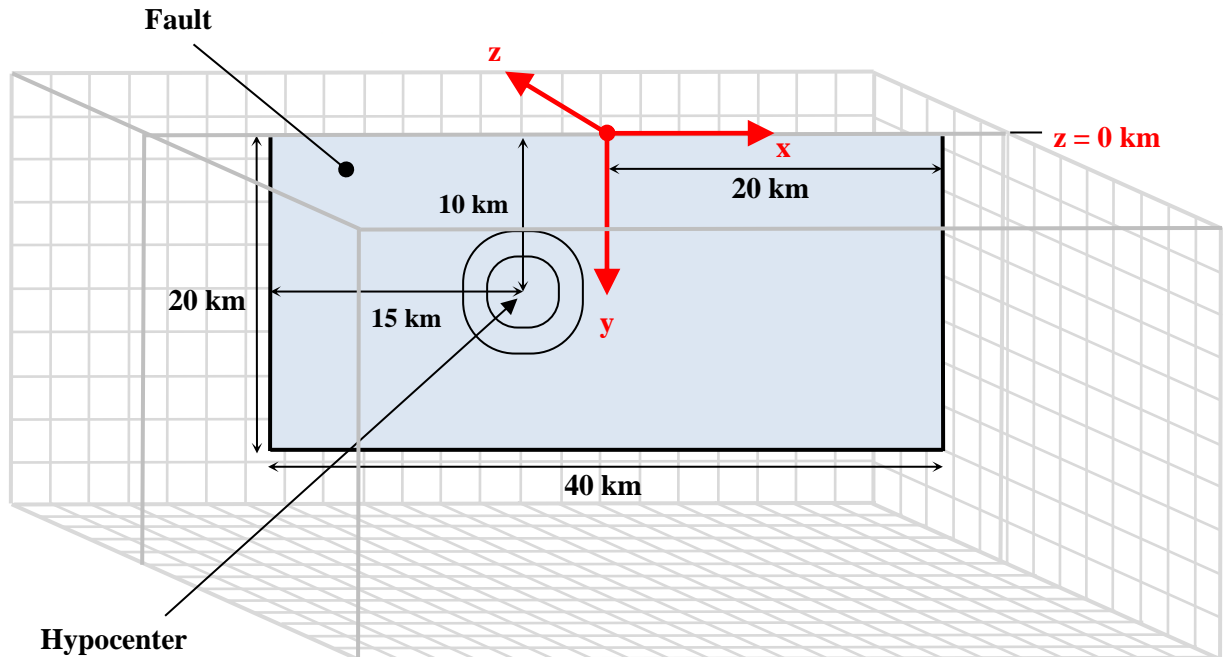
Part 1: Considerations for Plastic Benchmarks

Benchmarks with plasticity are more complicated to set up than benchmarks with linear elastic material properties. The following list highlights some special considerations to keep in mind when implementing a benchmark with off-fault plastic yielding.

- **Benchmark TPV27 uses non-associative Drucker-Prager viscoplasticity with yielding in shear.** Benchmark TPV26 uses linear elasticity. We include a detailed description of Drucker-Prager viscoplasticity.
- **The initial stress tensor is specified everywhere throughout the entire model volume.** Benchmarks with linear elasticity only need to specify initial shear and normal stresses on the fault surfaces. In TPV26 and TPV27, the initial shear and normal stresses on the faults are implied by resolving the initial stress tensor onto the fault surface.
- **Gravity must be accounted for.** The initial stress increases with depth, so gravitational acceleration is required to balance the gradient in the initial stress tensor.
- **You may need to apply traction forces to the boundary of the model volume.** The initial stresses exert traction forces on the boundary of the model volume. Depending on your implementation, you may need to balance them with externally-applied tractions.
- **Fluid pressure is included in the model.** Fluid pressure is hydrostatic, with water table at the earth's surface. The fluid pressure enters into the plastic constitutive law, and the friction law.
- **Nucleation is done using forced rupture.** Benchmarks with linear elasticity can simply impose higher shear stress near the hypocenter, but that is not possible with plasticity because it would be inconsistent with the initial stress tensor. In TPV26 and TPV27, we nucleate by using a zone of forced rupture surrounding the hypocenter. A detailed description is provided.

TPV26 and TPV27 use a linear slip-weakening friction law.

Part 2: Fault Geometry for TPV26 and TPV27



The model volume is a half-space.

The fault is a rectangle measuring 40000 m along-strike and 20000 m deep. The fault is a vertical, planar, strike-slip, right-lateral fault. The fault reaches the earth's surface.

The hypocenter is located 15 km from the left edge of the fault, at a depth of 10 km.

Introduce an (x, y, z) coordinate system, where x increases from left to right, y increases from top to bottom, and z increases from front to back. The origin is on the earth's surface, 20000 m from the right edge of the fault. In this coordinate system, the fault is:

$$\begin{aligned} -20000 \text{ m} &\leq x \leq 20000 \text{ m} \\ 0 \text{ m} &\leq y \leq 20000 \text{ m} \\ z &= 0 \text{ m} \end{aligned}$$

Part 3: Description of the 3D Benchmarks

Fault Geometry

Fault geometry for TPV26 and TPV27 is shown in part 2.

The hypocenter is 15000 m from the left edge of the fault, and 10000 m deep, at location $(x, y, z) = (-5000 \text{ m}, 10000 \text{ m}, 0 \text{ m})$.

Slip goes to zero at the border of a fault. So, a node which lies precisely on the border of a fault should not be permitted to slip. The free surface is not considered to be a border of the fault.

Material Properties

In TPV26, the entire model volume is a linear elastic material, with the following parameters.

$$\text{Density } \rho = 2670 \text{ kg/m}^3$$

$$\text{Shear-wave velocity } V_s = 3464 \text{ m/s}$$

$$\text{Pressure-wave velocity } V_p = 6000 \text{ m/s}$$

In TPV27, the entire model volume is a non-associative Drucker-Prager viscoplastic material that yields in shear, with the following parameters. Drucker-Prager viscoplasticity is described later, in part 6.

$$\text{Density } \rho = 2670 \text{ kg/m}^3$$

$$\text{Shear-wave velocity } V_s = 3464 \text{ m/s}$$

$$\text{Pressure-wave velocity } V_p = 6000 \text{ m/s}$$

$$\text{Cohesion } c = 1.36 \text{ MPa}$$

$$\text{Bulk friction } \nu = 0.1934$$

$$\text{Viscoplastic relaxation time } T_v = 0.03 \text{ s}$$

The material properties are the only difference between the elastic benchmark (TPV26), and the viscoplastic benchmark (TPV27).

Initial Stress Tensor

In an inelastic calculation, the initial stress tensor needs to be specified throughout the medium. The total (time-varying) stress, which equals the initial stress plus the stress change, is used in the plastic yield calculation. The (time-varying) normal and shear stresses acting on the fault surface are implied by the stress tensor in the adjacent rock.

For TPV26 and TPV27, the initial stress tensor varies only with depth. The components of the stress tensor are defined in the following table.

Stress Tensor Components	
<i>Component</i>	<i>Definition</i>
σ_{22}	Compressive stress in the vertical direction. Negative values denote compression.
σ_{11}	Compressive stress in the direction parallel to the fault. Negative values denote compression.
σ_{33}	Compressive stress in the direction perpendicular to the fault. Negative values denote compression. This equals the negative of the total normal stress on the fault.
σ_{13}	Shear stress in the horizontal plane. Positive values denote right-lateral shear stress on the fault. This equals the horizontal shear stress on the fault.
σ_{23}	Shear stress in a vertical plane perpendicular to the fault.
σ_{12}	Shear stress in a vertical plane parallel to the fault.

The stress tensor components can vary with time during the simulation.

Fluid pressure P_f is hydrostatic with water table at the surface. It varies with depth, but is constant in time.

For these 3D benchmarks, the fluid pressure and the initial values of the stress tensor are:

$$P_f = (1000 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(\text{depth in meters})$$

$$\sigma_{22} = -(2670 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(\text{depth in meters})$$

$$\sigma_{11} = \Omega(\text{depth})(b_{11} (\sigma_{22} + P_f) - P_f) + (1 - \Omega(\text{depth}))\sigma_{22}$$

$$\sigma_{33} = \Omega(\text{depth})(b_{33} (\sigma_{22} + P_f) - P_f) + (1 - \Omega(\text{depth}))\sigma_{22}$$

$$\sigma_{13} = \Omega(\text{depth}) \left(b_{13} (\sigma_{22} + P_f) \right)$$

$$\sigma_{23} = 0$$

$$\sigma_{12} = 0$$

The tapering coefficient $\Omega(\text{depth})$ causes the deviatoric component of stress to taper down to zero at depths between 15000 m and 20000 m.

$$\Omega(\text{depth}) = \begin{cases} 1, & \text{if depth} \leq 15000 \text{ m} \\ (20000 \text{ m} - \text{depth}) / (5000 \text{ m}), & \text{if } 15000 \text{ m} \leq \text{depth} \leq 20000 \text{ m} \\ 0, & \text{if depth} \geq 20000 \text{ m} \end{cases}$$

The coefficients b_{11} , b_{33} , and b_{13} are given in the following table.

Coefficient	Value for TPV26 and TPV27
b_{11}	0.926793
b_{33}	1.073206
b_{13}	-0.169029

Note that we have chosen the gravitational acceleration g to be *exactly* 9.8 m/s². Do not substitute a “more accurate” value of g .

Note that the initial axis of intermediate compressive stress is vertical, and its initial value is lithostatic. The initial axes of maximum and minimum compressive stress are horizontal, and their average value is lithostatic. For TPV27, the initial deviatoric stress at hypocenter depth is 93% of the plastic yield stress.

Note that we defined the stress tensor components σ_{ij} to be the **total** stress tensor, which includes both elastic stress and fluid pressure. To obtain the **effective** stress tensor, which includes only elastic stress, you need to add the fluid pressure P_f to the diagonal components. This can be shown as:

$$\text{total stress tensor} = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{12} & \sigma_{22} & \sigma_{23} \\ \sigma_{13} & \sigma_{23} & \sigma_{33} \end{pmatrix}$$

$$\text{effective stress tensor} = \begin{pmatrix} \sigma_{11} + P_f & \sigma_{12} & \sigma_{13} \\ \sigma_{12} & \sigma_{22} + P_f & \sigma_{23} \\ \sigma_{13} & \sigma_{23} & \sigma_{33} + P_f \end{pmatrix}$$

Friction Parameters and Nucleation

We use a linear slip-weakening friction law, which has the following six parameters.

Friction Parameters		
<i>Symbol</i>	<i>Parameter</i>	<i>Unit</i>
μ_s	Static coefficient of friction.	Dimensionless
μ_d	Dynamic coefficient of friction.	Dimensionless
d_0	Slip-weakening critical distance.	Meter
C_0	Frictional cohesion.	Pascal
T	Time of forced rupture.	Second
t_0	Forced rupture decay time	Second

The operation of the slip-weakening friction law is described in detail later, in part 4.

The friction parameter values are as follows, where r denotes distance to the hypocenter:

$$\mu_s = 0.18$$

$$\mu_d = 0.12$$

$$d_0 = 0.30 \text{ m}$$

$$C_0 = \begin{cases} 0.40 \text{ MPa} + (0.00072 \text{ MPa/m})(5000 \text{ m} - \text{depth}), & \text{if depth} \leq 5000 \text{ m} \\ 0.40 \text{ MPa}, & \text{if depth} \geq 5000 \text{ m} \end{cases}$$

$$r_{\text{crit}} = 4000 \text{ m}$$

$$T = \begin{cases} \frac{r}{0.7 V_S} + \frac{0.081 r_{\text{crit}}}{0.7 V_S} \left(\frac{1}{1 - (r/r_{\text{crit}})^2} - 1 \right), & \text{if } r < r_{\text{crit}} \\ 1.0\text{E}+9, & \text{if } r \geq r_{\text{crit}} \end{cases}$$

$$t_0 = 0.50 \text{ s}$$

The variation in the value of T near the hypocenter causes the rupture to nucleate, in a manner that is described later, in part 5.

The frictional cohesion C_0 (not to be confused with the plastic cohesion c) is 4.00 MPa at the earth's surface. It is 0.40 MPa at depths greater than 5000 m, and is linearly tapered in the uppermost 5000 m.

Running Time, Node Spacing, and Results

Run the model for times from **0.0 to 13.0 seconds after nucleation**.

Please submit results for two resolutions:

- Using **100 m node spacing** on the fault plane.
- Using **50 m node spacing** on the fault plane.

If you are unable to run the simulation with 50 m node spacing, then it is OK to omit the 50 m case. However, we strongly suggest that you run the simulation with 50 m resolution if at all possible.

The requested output files are:

- **On-fault time-series files**, which give slips, slip rates, and stresses for each on-fault station at each time step. These files are described in part 8.
- **Off-fault time-series files**, which give displacements and velocities for each off-fault station at each time step. These files are described in part 9.
- **A contour-plot file** which, for each node on the fault, gives the time at which the slip rate first changes from 0 to greater than 0.001 m/s. This file is described in part 10.

Part 4: Linear Slip-Weakening Friction

Benchmarks TPV26 and TPV27 use linear slip-weakening friction, with frictional cohesion and forced rupture. This friction law has the following parameters and variables:

Friction Parameters		
<i>Symbol</i>	<i>Parameter</i>	<i>Unit</i>
μ_s	Static coefficient of friction.	Dimensionless
μ_d	Dynamic coefficient of friction.	Dimensionless
d_0	Slip-weakening critical distance.	Meter
C_0	Frictional cohesion.	Pascal
T	Time of forced rupture.	Second
t_0	Forced rupture decay time	Second

Friction Variables		
<i>Symbol</i>	<i>Parameter</i>	<i>Unit</i>
σ_n	Total normal stress acting on the fault, taken to be positive in compression.	Pascal
τ	Shear stress acting on the fault.	Pascal
P_f	Fluid pressure (which is assumed to be constant in time).	Pascal
$\sigma_n - P_f$	Effective normal stress acting on the fault, which is positive in compression.	Pascal

When the fault is sliding, the shear stress τ at a given point on the fault is given by:

$$\tau = C_0 + \mu \max(0, \sigma_n - P_f)$$

The time-varying coefficient of friction μ is given by the following formulas, where D is the total distance the node has slipped, and t is the time since the start of the simulation:

$$\mu = \mu_s + (\mu_d - \mu_s) \max(f_1, f_2)$$

$$f_1 = \begin{cases} D/d_0, & \text{if } D < d_0 \\ 1, & \text{if } D \geq d_0 \end{cases}$$

$$f_2 = \begin{cases} 0, & \text{if } t < T \\ (t - T)/t_0, & \text{if } T \leq t < T + t_0 \\ 1, & \text{if } t \geq T + t_0 \end{cases}$$

The distance D that the node has slipped is path-integrated. For example, if the node slips 0.4 m in one direction and then 0.1 m in the opposite direction, the value of D is 0.5 m (and not 0.3 m).

The time of forced rupture T is used to nucleate the rupture, as described in part 5. At time $t = T$, the coefficient of friction begins to decrease, reaching its final dynamic value at time $t = T + t_0$. So, the node is forced to begin sliding somewhere in the interval $T < t < T + t_0$. Notice that because of the expression $\max(f_1, f_2)$, forcing has no effect if friction has already weakened due to the accumulated slip. If a given node does not undergo forced rupture, then the value of T is 1.0E9.

Tension on the fault: If you encounter tension on the fault, you should **treat tension on the fault the same as if the effective normal stress equals zero**. This is shown in the above formulas by the expression $\max(0, \sigma_n - P_f)$.

You should **constrain the motion of the node so that the fault cannot open (that is, only permit sliding parallel to the fault), even when the fault is in tension**. During the time the fault is in tension, continue to accumulate the slip distance D as usual.

In your time-series files, where you are asked to report “normal stress,” you should report the effective normal stress which is defined to be $\sigma_n - P_f$.

Part 5: Nucleation

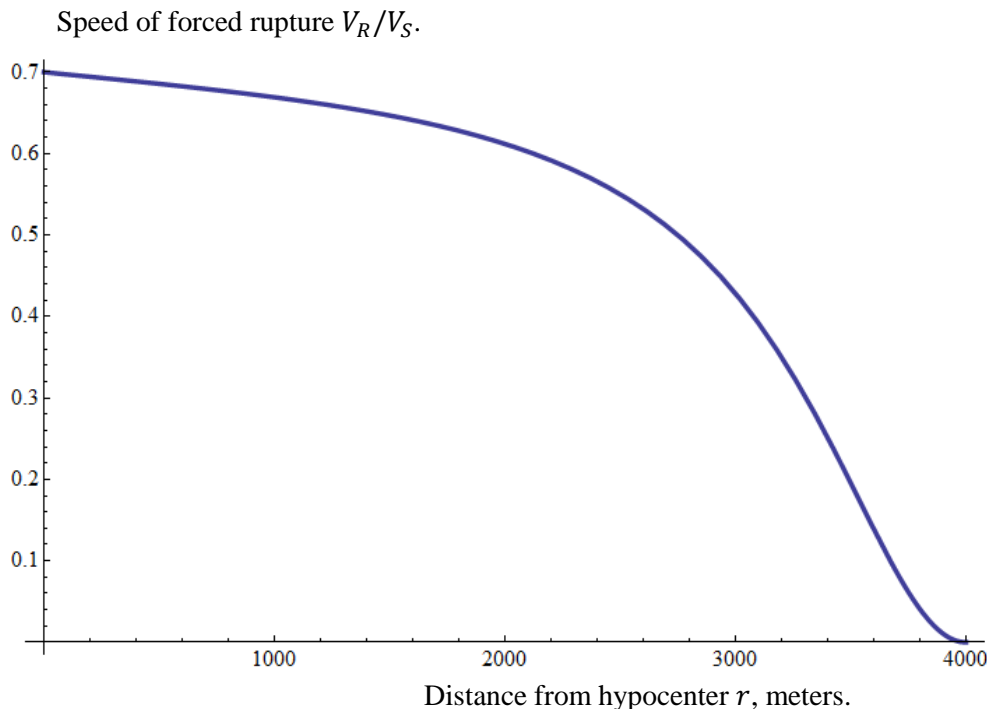
Nucleation is performed by forcing the fault to rupture, within a circular zone surrounding the hypocenter. Forced rupture is achieved by artificially reducing the friction coefficient, beginning at a specified time T . The parameter t_0 specifies how long it takes for the friction coefficient to be artificially reduced from its static value to its dynamic value. So, the friction coefficient reaches its dynamic value at time $T + t_0$. We reduce the friction coefficient gradually, over an interval of time, in order to smooth the nucleation process and reduce unwanted oscillations.

Recall that we selected $t_0 = 0.5$ s, and that the time of forced rupture is defined to be

$$T = \begin{cases} \frac{r}{0.7 V_S} + \frac{0.081 r_{\text{crit}}}{0.7 V_S} \left(\frac{1}{1 - (r/r_{\text{crit}})^2} - 1 \right), & \text{if } r < r_{\text{crit}} \\ 1.0\text{E}+9, & \text{if } r \geq r_{\text{crit}} \end{cases}$$

where r is distance from the hypocenter, and where $r_{\text{crit}} = 4000$ m is the radius of the nucleation zone. At the hypocenter, $T = 0$. The value of T then increases with distance from the hypocenter, which creates an expanding circular region of forced rupture.

The time T is computed so that the forced rupture expands at a variable speed. Near the hypocenter, the forced rupture expands at a speed of $0.7 V_S$. The speed decreases with increasing distance from the hypocenter, finally reaching a speed of zero at a distance of 4000 m from the hypocenter. The variable speed allows for a smooth transition between forced rupture and spontaneous rupture, because the spontaneous rupture should gradually overtake the ever-slowing forced rupture. The graph below shows the forced rupture speed $V_R \equiv dr/dT$ as a function of distance from the hypocenter.



Part 6: Drucker-Prager Viscoplasticity

Constitutive Law

For TPV27 we are using non-associative Drucker-Prager viscoplasticity with yielding in shear. The material is characterized by seven constitutive parameters:

$$\text{Density } \rho = 2670 \text{ kg/m}^3$$

$$\text{Shear-wave velocity } V_s = 3464 \text{ m/s}$$

$$\text{Pressure-wave velocity } V_p = 6000 \text{ m/s}$$

$$\text{Cohesion } c = 1.36 \text{ MPa}$$

$$\text{Bulk friction } \nu = 0.1934$$

$$\text{Viscoplastic relaxation time } T_v = 0.03 \text{ s}$$

$$\text{Fluid pressure } P_f = (1000 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(\text{depth in meters})$$

Except for the fluid pressure, all the constitutive parameters are uniform. The “cohesion” listed here is not to be confused with the “frictional cohesion” that appears in the fault friction law. From these six constitutive parameters, we can derive several additional ones:

$$\text{Shear modulus } \mu = \rho V_s^2$$

$$\text{Lame volumetric parameter } \lambda = \rho(V_p^2 - 2V_s^2)$$

$$\text{Bulk modulus } K = \lambda + \frac{2}{3}\mu$$

$$\text{Angle of friction } \phi = \tan^{-1} \nu$$

Let σ_{ij} denote the stress tensor. Define:

$$\text{Mean stress } \sigma_m = \frac{1}{3}(\sigma_{11} + \sigma_{22} + \sigma_{33})$$

$$\text{Stress deviator } s_{ij} = \sigma_{ij} - \sigma_m \delta_{ij}$$

$$\text{Second invariant of the stress deviator } J_2(\sigma) = \frac{1}{2} \sum_{i,j} s_{ij} s_{ji}$$

$$\text{Drucker-Prager yield stress } Y(\sigma) = \max(0, c \cos \phi - (\sigma_m + P_f) \sin \phi)$$

$$\text{Drucker-Prager yield function } F(\sigma) = \sqrt{J_2(\sigma)} - Y(\sigma)$$

$$\text{Drucker-Prager plastic projection operator } P(\sigma) = \sigma^P \text{ where}$$

$$\sigma_{ij}^P = \begin{cases} \sigma_{ij}, & \text{if } F(\sigma) \leq 0 \\ \sigma_m \delta_{ij} + \left(Y(\sigma) / \sqrt{J_2(\sigma)} \right) s_{ij}, & \text{if } F(\sigma) \geq 0 \end{cases}$$

Let ϵ_{ij} denote the strain tensor, let ϵ_{ij}^{vp} denote the viscoplastic strain tensor, and let ϵ_{ij}^e denote the elastic strain tensor. Then the material constitutive equations are:

$$\epsilon_{ij}^e = \epsilon_{ij} - \epsilon_{ij}^{vp}$$

$$\sigma_{ij} = \lambda(\epsilon_{11}^e + \epsilon_{22}^e + \epsilon_{33}^e)\delta_{ij} + 2\mu\epsilon_{ij}^e$$

$$\frac{d}{dt} \left(\lambda(\epsilon_{11}^{vp} + \epsilon_{22}^{vp} + \epsilon_{33}^{vp})\delta_{ij} + 2\mu\epsilon_{ij}^{vp} \right) = \frac{1}{T_v} (\sigma_{ij} - \sigma_{ij}^P)$$

(Remarks: In writing these equations, we implicitly assume that the strain tensor ϵ_{ij} is a known function of time, and we want to solve for the stress tensor σ_{ij} as a function of time. The first equation is the definition of the viscoplastic strain tensor ϵ_{ij}^{vp} , and the second equation is the definition of the elastic strain tensor ϵ_{ij}^e . Then the third equation is a set of coupled differential equations which are to be solved for σ_{ij} . The third equation can be simplified, because for our choice of $P(\sigma)$ it can be shown that $\epsilon_{11}^{vp} + \epsilon_{22}^{vp} + \epsilon_{33}^{vp} = 0$. We retain the term because we prefer to write the constitutive equations in a form that is valid for any choice of $P(\sigma)$.)

When $F(\sigma) < 0$, the material behaves like a linear elastic material, with Lamé parameters λ and μ . The viscoplastic strain tensor ϵ_{ij}^{vp} remains constant while the material is behaving elastically.

When $F(\sigma) \geq 0$, the material *yields viscoplastically*. Viscoplastic yielding can be understood as follows:

- If the material is subjected to an instantaneous (or very rapid) increment of strain, then it responds like a linear elastic material, with Lamé parameters λ and μ . The viscoplastic strain tensor ϵ_{ij}^{vp} remains unchanged.
- If time passes with no change in the strain tensor, then the stress tensor σ_{ij} decays exponentially to the value σ_{ij}^P , with characteristic time T_v . The change in the stress tensor is accompanied by a corresponding change in the viscoplastic strain tensor ϵ_{ij}^{vp} . We say that the material *yields in shear* because the stress tensor σ_{ij} changes by an amount proportional to the stress deviator s_{ij} , and the viscoplastic strain tensor ϵ_{ij}^{vp} is purely deviatoric (*i.e.*, has zero trace).

Technical remarks: We call P a *projection operator* because $P(P(\sigma)) = P(\sigma)$. When viscoplasticity is described by a projection operator, it is called a *Duvaut-Lions model*. We chose P so that $\sigma - P(\sigma)$ is purely deviatoric, and is proportional to the stress deviator s . The *yield cone* is the set $\{\sigma \mid F(\sigma) \leq 0\}$. We say the material is *non-associative* because $P(\sigma)$ is not the point on yield cone that is closest to σ .

Return Map Algorithm

The return map algorithm is a popular algorithm for implementing viscoplasticity, so we describe it here.

Given:

σ_{ij}^0 = Stress tensor at the start of a time step.

ϵ_{ij} = Strain tensor increment applied during the time step.

Δt = Duration of the time step.

We want to find:

σ_{ij} = Stress tensor at the end of the time step.

Step 1. Calculate a *trial stress tensor* by assuming that the material behaves elastically:

$$\sigma_{ij}^{\text{trial}} = \sigma_{ij}^0 + \lambda(\epsilon_{11} + \epsilon_{22} + \epsilon_{33})\delta_{ij} + 2\mu\epsilon_{ij}$$

Step 2. Calculate the following:

$$\text{Trial mean stress } \sigma_m^{\text{trial}} = \frac{1}{3}(\sigma_{11}^{\text{trial}} + \sigma_{22}^{\text{trial}} + \sigma_{33}^{\text{trial}})$$

$$\text{Trial stress deviator } s_{ij}^{\text{trial}} = \sigma_{ij}^{\text{trial}} - \sigma_m^{\text{trial}}\delta_{ij}$$

$$\text{Second invariant of the trial stress deviator } J_2(\sigma^{\text{trial}}) = \frac{1}{2}\sum_{i,j} s_{ij}^{\text{trial}} s_{ji}^{\text{trial}}$$

$$\text{Trial Drucker-Prager yield stress } Y(\sigma^{\text{trial}}) = \max(0, c \cos \phi - (\sigma_m^{\text{trial}} + P_f) \sin \phi)$$

$$\text{Trial Drucker-Prager yield function } F(\sigma^{\text{trial}}) = \sqrt{J_2(\sigma^{\text{trial}})} - Y(\sigma^{\text{trial}})$$

Step 3. If $F(\sigma^{\text{trial}}) \leq 0$, then this is an *elastic step*. In this case, we take:

$$\sigma_{ij} = \sigma_{ij}^{\text{trial}}$$

Step 4. If $F(\sigma^{\text{trial}}) > 0$, then this is a *viscoplastic step*. In this case, we take:

$$r = \exp(-\Delta t/T_v) + (1 - \exp(-\Delta t/T_v)) Y(\sigma^{\text{trial}}) / \sqrt{J_2(\sigma^{\text{trial}})}$$

$$\sigma_{ij} = \sigma_m^{\text{trial}}\delta_{ij} + r s_{ij}^{\text{trial}}$$

Sample Fortran Code

Courtesy of Joe Andrews and Michael Barall, here is sample Fortran code for the return map algorithm.

```
c ---[ Constitutive parameters ]---
c   Density
c   density = 2670.0
c   Vs
c   vs = 3464.0
c   Vp
c   vp = 6000.0
c   Shear modulus
c   mu = density*(vs**2)
c   Lamé volumetric parameter
c   lambda = density*(vp**2 - 2.0*(vs**2))
c   Cohesion is 1.36 MPa
c   cohes = 1.36e6
c   Bulk friction
c   blkfric = 0.1934
c   Angle of friction
c   angfric = atan(blkfric)
c   Viscoplastic relaxation time
c   tvisc = 0.03
c   Fluid pressure, depth-dependent
c   fluidpresh = 1000.0*9.8*depth

c ---[ Calculate trial stress tensor ]---
c   Stress tensor components: sxx,syy,szz,sxy,syz,sxz
c   Strain tensor increment components: exx,eyy,ezz,exy,eyz,exz
c   etrace = exx + eyy + ezz
c   sxx = sxx + lambda*etrace + 2.0*mu*exx
c   syy = syy + lambda*etrace + 2.0*mu*eyy
c   szz = szz + lambda*etrace + 2.0*mu*ezz
c   sxy = sxy + 2.0*mu*exy
c   syz = syz + 2.0*mu*eyz
c   sxz = sxz + 2.0*mu*exz

c ---[ Calculate trial yield stress ]---
c   Mean stress
c   sm = (sxx + syy + szz)/3.0
c   Find stress deviator components
c   sdx = sxx - sm
c   sdy = syy - sm
c   sdz = szz - sm
c   sdx = sxy
```



```

    sdyz = syz
    sdxz = sxz
c    Second invariant of stress deviator
    secinv = 0.5*(sdxx**2+sdyy**2+sdzz**2)+sdxy**2+sdyz**2+sdxz**2
c    Scalar measure of shear stress
    tau = sqrt(secinv)
c    Yield stress
    taulim = cohes*cos(angfric) - (sm + fluidpresh)*sin(angfric)
    taulim = max(0.0, taulim)

c ---[ Yielding in shear ]---
c    Duration of time step: deltat
c    Stress deviators are adjusted, no change to mean stress
    if (tau .gt. taulim) then
        decay = exp(-deltat/tvisc)
        yldfac = decay + (1.0-decay)*taulim/tau
        sxx = sdxx*yldfac + sm
        syy = sdyy*yldfac + sm
        szz = sdzz*yldfac + sm
        sxy = sdxy*yldfac
        syz = sdyz*yldfac
        sxz = sdxz*yldfac
    end if

```

Part 7: Methods for Implementing Gravity and Boundary Traction

In an inelastic calculation, the initial stress needs to be specified throughout the medium. The total stress, which equals initial stress plus stress change, is used in the yield calculation. This creates two challenges.

- Because the initial stress varies with depth, a gravitational force is required to balance the gradient of the initial stress tensor.
- The initial stress tensor exerts traction forces on the boundary of the mesh. If the code permits the mesh boundaries to move, then external tractions must be applied to balance these forces.

There are two basic methods for dealing with gravity and boundary tractions, described below. You may choose either method.

Method 1 — Using Stress Change

The stress tensor that is calculated and stored in the code is the change of stress from some initial state. The initial state is assumed to be in static equilibrium. With this method, the gravity and boundary tractions do not appear explicitly in the calculation, because as far as the code is concerned, all the initial stresses and forces are equal to zero.

In an elastic calculation, the initial state does not need to be known. In an inelastic calculation, the initial stress tensor must be specified and stored within each element (it varies only with depth in this case). Performing the plastic yield calculation then requires three steps:

1. Add the stress change to the initial stress, obtaining the total stress tensor.
2. Perform the plastic yield calculation (for example, with the return map algorithm), obtaining a new total stress tensor.
3. Subtract the initial stress from the new total stress tensor, obtaining a new stress change.

Because the code starts off with all stresses and forces equal to zero, you must “manually” specify the initial shear stress and initial effective normal stress at each node on the fault surface. To calculate the frictional sliding on the fault, you must (a) calculate the tractions on the fault surface implied by the change in stress, and then (b) add the initial shear stress and initial effective normal stress, to obtain the total shear stress and total effective normal stress which appear in the friction law.

Method 2 — Using Explicit Gravity and Boundary Traction

The stress tensor that is calculated and stored in the code is the total stress tensor. The stress tensor throughout the medium must be initialized to a state that is in static equilibrium. The equilibrium stress state might come from a static calculation. In this case, because it depends only on depth, the initial stress state is simple enough to specify directly.

The acceleration of gravity must be added explicitly to the equations of motion. The gradient of the initial stress tensor is balanced by gravitational forces. For TPV26 and TPV27, you should set the gravitational acceleration g to be exactly 9.8 m/s^2 .

If your code allows the mesh boundaries to move, then you must also apply external tractions to the mesh boundary, to balance the tractions caused by the initial stress. You need to calculate and store the force exerted on each boundary node by the initial stress conditions (including the gravitational force). Then, during the dynamic calculation, apply the negative of the stored force to each node lying in the boundary, thereby eliminating unwanted motion of the boundary. There are two ways to calculate the forces:

1. Perform a preliminary static calculation.
2. Analytically calculate the forces, in a manner that depends on the details of your code.

Because of approximation errors in the equations of motion, there may be small velocities throughout the mesh before real signals arrive. They are caused by small residual forces that exist because the nodal forces calculated from the initial stress tensor may not precisely cancel out the calculated gravitational forces. This effect is generally small enough to be ignored.

(Remark: If you do a preliminary static calculation to obtain boundary tractions, you can at the same time calculate the residual forces. Then, during the dynamic calculation, you can cancel out both the boundary tractions and the residual forces, thereby eliminating the small velocities. Just be careful that you don't also cancel out the traction forces acting on the fault surface!)

The plastic yield calculation has the total stress tensor available, so that the initial stress tensor does not need to be stored.

The code automatically calculates the total shear stress and total normal stress acting on the fault surface. So, it is not necessary to “manually” specify initial stresses acting on the fault surface.

However, the frictional sliding calculation also requires the fluid pressure, so that effective normal stress can be computed. There are two ways you could obtain the fluid pressure at each node on the fault surface. You may use either technique.

1. Calculate the fluid pressure by applying the formula

$$P_f = (1000 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(\text{depth in meters})$$

This should work well for fault nodes not located on the earth's surface. However, special considerations apply to a fault node located at the earth's surface. For most codes it is incorrect to assign a fluid pressure of zero. We recommend that the fluid pressure be set as follows: (a) If your code has an initial stress tensor that is uniform within each element, then a fault node at the earth's surface should be assigned a fluid pressure corresponding to a depth of 1/2 element. (b) If

your code has an initial stress tensor that varies linearly with depth inside each element, then a fault node at the earth's surface should be assigned a fluid pressure corresponding to a depth of 1/3 element.

2. Perform a preliminary static calculation to obtain the traction exerted on the fault surface by fluid pressure. Set up the static calculation to use density and initial stress as

$$\rho = 1000 \text{ kg/m}^3$$
$$\sigma_{11} = \sigma_{22} = \sigma_{33} = -(1000 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(\text{depth in meters})$$

The static calculation should also include the gravitational force. You can numerically calculate the normal stress exerted at each node on the fault surface by fluid pressure, and store the calculated normal stress. Then, during the dynamic calculation, you use the stored value as the fluid pressure.

(Because of numerical approximation errors, the static calculation may also yield a small shear stress acting on the fault surface. If you wish, you may store the calculated shear stress, and deduct it from the total shear stress (using vector subtraction) during the dynamic calculation.)

(Remark: Some codes may prefer to calculate and store the effective stress tensor at each point of the medium, rather than the total stress tensor. This does not fundamentally change the method, although some details are changed. In particular, because resolving the effective stress tensor onto the fault surface yields effective normal stress, it is not necessary to separately calculate the fluid pressure on the fault surface.)

Part 8: On-Fault Stations, and Time-Series File Format

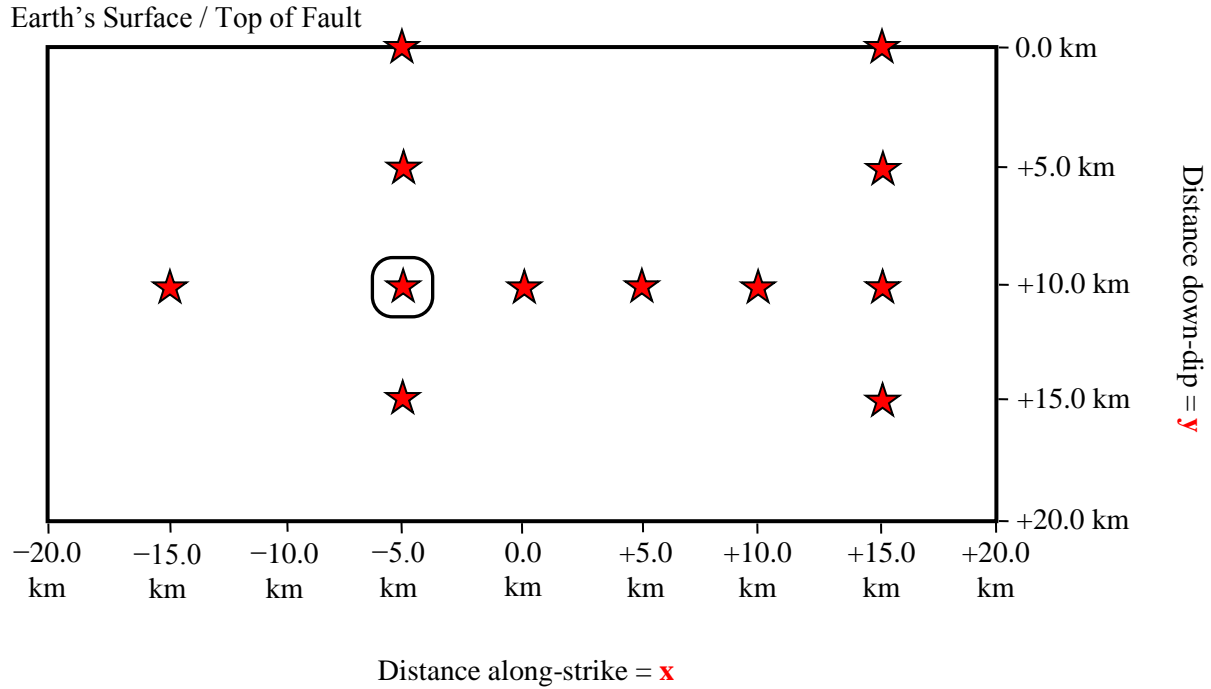
The benchmarks use 12 stations on the fault, which are listed below. A diagram of station locations is given following the table. You need to supply one time-series file for each station.

On-Fault Stations, for TPV26 and TPV27	
Station Name	Location
faultst-050dp000	On fault, -5.0 km along strike, 0 km down-dip.
faultst150dp000	On fault, 15.0 km along strike, 0 km down-dip.
faultst-050dp050	On fault, -5.0 km along strike, 5.0 km down-dip.
faultst150dp050	On fault, 15.0 km along strike, 5.0 km down-dip.
faultst-150dp100	On fault, -15.0 km along strike, 10.0 km down-dip.
faultst-050dp100	On fault, -5.0 km along strike, 10.0 km down-dip (hypocenter).
faultst000dp100	On fault, 0 km along strike, 10.0 km down-dip.
faultst050dp100	On fault, 5.0 km along strike, 10.0 km down-dip.
faultst100dp100	On fault, 10.0 km along strike, 10.0 km down-dip.
faultst150dp100	On fault, 15.0 km along strike, 10.0 km down-dip.
faultst-050dp150	On fault, -5.0 km along strike, 15.0 km down-dip.
faultst150dp150	On fault, 15.0 km along strike, 15.0 km down-dip.

If you do not have a node at the location of a station, there are two options: (1) you can move the station to the nearest node, or (2) you can interpolate the data values from nodes near the station location.

Note: Location along-strike is measured relative to the origin of the (x, y, z) coordinate system Positive locations are to the right of the origin.

Station Locations on the Fault



There are 12 stations:

- -5.0 km and +15.0 km along-strike, and 0 km down-dip distance.
- -5.0 km and +15.0 km along-strike, and 5.0 km down-dip distance.
- -15.0 km, -5.0 km, 0.0 km, +5.0 km, +10 km, and +15.0 km along-strike, and 10.0 km down-dip distance.
- -5.0 km and +15.0 km along-strike, and 15.0 km down-dip distance.

Note that location along-strike is measured relative to the origin of the (x, y, z) coordinate system. Positive locations are to the right of the origin.

Each time series file is an ASCII file that contains 8 data fields, as follows.

On-Fault Time Series Data Fields for TPV26 and TPV27	
Field Name	Description, Units, and Sign Convention
t	Time (s).
h-slip	Horizontal slip along-strike (m). Sign convention: Positive means right lateral slip.
h-slip-rate	Horizontal slip rate along-strike (m/s). Sign convention: Positive means right lateral motion.
h-shear-stress	Horizontal shear stress along-strike (MPa). Sign convention: Positive means shear stress that tends to cause right-lateral slip.
v-slip	Vertical along-dip slip (m). Sign convention: Positive means downward slip (that is, the far side of the fault moving downward relative to the near side of the fault).
v-slip-rate	Vertical along-dip slip rate (m/s). Sign convention: Positive means downward motion (that is, the far side of the fault moving downward relative to the near side of the fault).
v-shear-stress	Vertical along-dip shear stress (MPa). Sign convention: Positive means shear stress that tends to cause downward slip (that is, the far side of the fault moving downward relative to the near side of the fault).
n-stress	Normal stress (MPa). Note: You should report effective normal stress , which is defined to be total normal stress minus fluid pressure. Sign convention: Positive means extension .

The **near side** of a fault is in the front of the diagram (the $-z$ side of the fault).

The **far side** of a fault is in the back of the diagram (the $+z$ side of the fault).

The on-fault time series file consists of three sections, as follows.

On-Fault Time Series File Format for TPV26 and TPV27	
File Section	Description
File Header	<p>A series of lines, each beginning with a # symbol, that gives the following information:</p> <ul style="list-style-type: none"> • Benchmark problem (TPV26 or TPV27) • Author • Date • Code • Code version • Node spacing or element size • Time step • Number of time steps in file • Station location • Descriptions of data columns (7 lines) • Anything else you think is relevant
Field List	<p>A single line, which lists the names of the 8 data fields, in column order, separated by spaces. It should be:</p> <pre>t h-slip h-slip-rate h-shear-stress v-slip v-slip-rate v-shear-stress n-stress</pre> <p>(all on one line). The server examines this line to check that your file contains the correct data fields.</p>
Time History	<p>A series of lines. Each line contains 8 numbers, which give the data values for a single time step. The lines must appear in order of increasing time.</p> <p>C/C++ users: For all data fields except the time, we recommend using 14.6E or 14.6e floating-point format. For the time field, we recommend using 20.12E or 20.12e format (but see the note on the next page).</p> <p>Fortran users: For all data fields except the time, we recommend using E15.7 or 1PE15.6 floating-point format. For the time field, we recommend using E21.13 or 1PE21.12 format (but see the note on the next page).</p> <p>The server accepts most common numeric formats. If the server cannot understand your file, you will see an error message when you attempt to upload the file.</p>

Note: We recommend higher precision for the time field so the server can tell that your time steps are all equal. (If the server thinks your time steps are not all equal, it will refuse to apply digital filters to your data.) If you use a “simple” time step value like 0.01 seconds or 0.005 seconds, then there is no need for higher precision, and you can write the time using the same precision as all the other data fields. When you upload a file, the server will warn you if it thinks your time steps are not all equal.

Here is an example of an on-fault time-series file. This is an invented file, not real modeling data.

```
# Example on-fault time-series file.
#
# This is the file header:
# problem=TPV26
# author=A.Modeler
# date=2014/01/23
# code=MyCode
# code_version=3.7
# element_size=100 m
# time_step=0.008
# num_time_steps=1625
# location= on fault, 5.0 km along strike, 10.0km down-dip
# Column #1 = Time (s)
# Column #2 = horizontal slip (m)
# Column #3 = horizontal slip rate (m/s)
# Column #4 = horizontal shear stress (MPa)
# Column #5 = vertical slip (m)
# Column #6 = vertical slip rate (m/s)
# Column #7 = vertical shear stress (MPa)
# Column #8 = normal stress (MPa)
#
# The line below lists the names of the data fields:
t h-slip h-slip-rate h-shear-stress v-slip v-slip-rate v-shear-stress n-stress
#
# Here is the time-series data.
# There should be 8 numbers on each line, but this page is not wide enough
# to show 8 numbers on a line, so we only show the first five.
0.000000E+00  0.000000E+00  0.000000E+00  7.000000E+01  0.000000E+00  ...
5.000000E-03  0.000000E+00  0.000000E+00  7.104040E+01  0.000000E+00  ...
1.000000E-02  0.000000E+00  0.000000E+00  7.239080E+01  0.000000E+00  ...
1.500000E-02  0.000000E+00  0.000000E+00  7.349000E+01  0.000000E+00  ...
2.000000E-02  0.000000E+00  0.000000E+00  7.440870E+01  0.000000E+00  ...
2.500000E-02  0.000000E+00  0.000000E+00  7.598240E+01  0.000000E+00  ...
# ... and so on.
```

Part 9: Off-Fault Stations, and Time-Series File Format

The benchmarks use the 6 off-fault stations listed below. All stations are at the earth's surface.

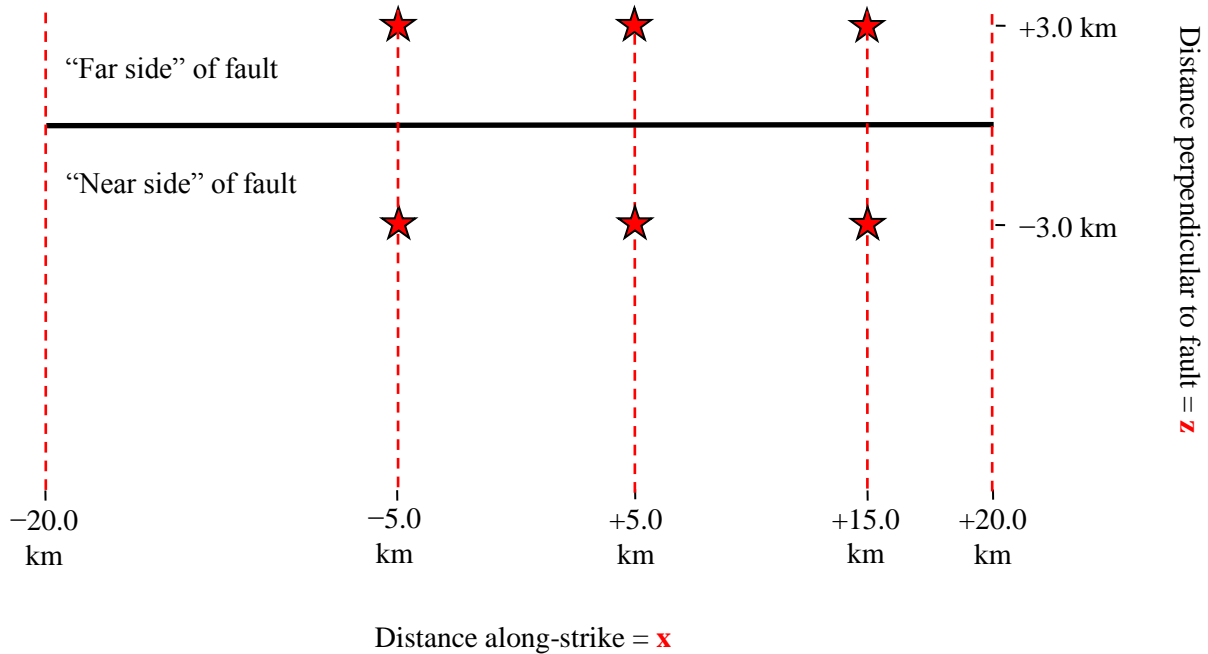
Refer to the next page for a diagram of station locations. You need to supply one time-series file for each station.

Off-Fault Stations for TPV26 and TPV27	
Station Name	Location
body030st-050dp000	3.0 km off fault (far side), -5.0 km along strike, 0 km depth.
body-030st-050dp000	-3.0 km off fault (near side), -5.0 km along strike, 0 km depth.
body030st050dp000	3.0 km off fault (far side), 5.0 km along strike, 0 km depth.
body-030st050dp000	-3.0 km off fault (near side), 5.0 km along strike, 0 km depth.
body030st150dp000	3.0 km off fault (far side), 15.0 km along strike, 0 km depth.
body-030st150dp000	-3.0 km off fault (near side), 15.0 km along strike, 0 km depth.

In the station names, the first number is the horizontal perpendicular distance from the station to the fault. A positive number means that the station is located on the **far side** of the fault.

If you do not have a node at the location of a station, there are two options: (1) you can move the station to the nearest node, or (2) you can interpolate the data values from nodes near the station location.

Off-Fault Station Locations



The diagram shows the earth's surface, looking downwards.

There are 6 stations at the earth's surface:

- -5.0 km, +5.0 km, and +15.0 km along-strike, and +3.0 km perpendicular distance from the fault trace.
- -5.0 km, +5.0 km, and +15.0 km along-strike, and -3.0 km perpendicular distance from the fault trace.

The **near side** of the fault is in the front of the diagram (the $-z$ side of the fault).

The **far side** of the fault is in the back of the diagram (the $+z$ side of the fault).

Positive perpendicular distance from the fault means that the station is on the **far side**.

Each time series file is an ASCII file that contains 7 data fields, as follows.

Off-Fault Time Series Data Fields for TPV26 and TPV27	
Field Name	Description, Units, and Sign Convention
t	Time (s).
h-disp	Horizontal displacement, parallel to the fault strike (m). Sign convention: Positive means displacement to the right relative to the station's initial position (that is, in the $+x$ direction).
h-vel	Horizontal velocity, parallel to the fault strike (m/s). Sign convention: Positive means motion to the right (that is, in the $+x$ direction).
v-disp	Vertical displacement (m). Sign convention: Positive means displacement downward relative to the station's initial position (that is, in the $+y$ direction).
v-vel	Vertical velocity (m/s). Sign convention: Positive means motion downward (that is, in the $+y$ direction).
n-disp	Horizontal displacement, perpendicular to the fault strike (m). Sign convention: Positive means displacement away from the viewer, into the paper (away from near side of the fault and toward the far side of the fault) relative to the station's initial position. In other words, displacement in the $+z$ direction.
n-vel	Horizontal velocity, perpendicular to the fault strike (m/s). Sign convention: Positive means motion away from the viewer, into the paper (that is, away from near side of the fault and toward the far side of the fault). In other words, motion in the $+z$ direction.

The **near side** of the fault is in the front of the diagram (the $-z$ side of the fault).

The **far side** of the fault is in the back of the diagram (the $+z$ side of the fault).

The off-fault time series file consists of three sections, as follows.

Off-Fault Time Series File Format for TPV26 and TPV27	
File Section	Description
File Header	<p>A series of lines, each beginning with a # symbol, that gives the following information:</p> <ul style="list-style-type: none"> • Benchmark problem (TPV26 or TPV27) • Author • Date • Code • Code version • Node spacing or element size • Time step • Number of time steps in file • Station location • Descriptions of data columns (7 lines) • Anything else you think is relevant
Field List	<p>A single line, which lists the names of the 7 data fields, in column order, separated by spaces. It should be:</p> <pre>t h-disp h-vel v-disp v-vel n-disp n-vel</pre> <p>(all on one line). The server examines this line to check that your file contains the correct data fields.</p>
Time History	<p>A series of lines. Each line contains 7 numbers, which give the data values for a single time step. The lines must appear in order of increasing time.</p> <p>C/C++ users: For all data fields except the time, we recommend using 14.6E or 14.6e floating-point format. For the time field, we recommend using 20.12E or 20.12e format (but see the note on the next page).</p> <p>Fortran users: For all data fields except the time, we recommend using E15.7 or 1PE15.6 floating-point format. For the time field, we recommend using E21.13 or 1PE21.12 format (but see the note on the next page).</p> <p>The server accepts most common numeric formats. If the server cannot understand your file, you will see an error message when you attempt to upload the file.</p>

Note: We recommend higher precision for the time field so the server can tell that your time steps are all equal. (If the server thinks your time steps are not all equal, it will refuse to apply digital filters to your data.) If you use a “simple” time step value like 0.01 seconds or 0.005 seconds, then there is no need for higher precision, and you can write the time using the same precision as all the other data fields. When you upload a file, the server will warn you if it thinks your time steps are not all equal.

Here is an example of an off-fault time-series file. This is an invented file, not real modeling data.

```
# Example off-fault time-series file.
#
# This is the file header:
# problem=TPV26
# author=A.Modeler
# date=2014/01/23
# code=MyCode
# code_version=3.7
# element_size=100 m
# time_step=0.008
# num_time_steps=1625
# location= 3.0 km off fault, 5.0 km along strike, 0.0 km depth
# Column #1 = Time (s)
# Column #2 = horizontal displacement (m)
# Column #3 = horizontal velocity (m/s)
# Column #4 = vertical displacement (m)
# Column #5 = vertical velocity (m/s)
# Column #6 = normal displacement (m)
# Column #7 = normal velocity (m/s)
#
# The line below lists the names of the data fields:
t h-disp h-vel v-disp v-vel n-disp n-vel
#
# Here is the time-series data.
# There should be 7 numbers on each line, but this page is not wide enough
# to show 7 numbers on a line, so we only show the first five.
0.000000E+00 0.000000E+00 0.000000E+00 0.000000E+00 0.000000E+00 ...
5.000000E-03 -2.077270E-85 -2.575055E-83 -2.922774E-86 -3.623018E-84 ...
1.000000E-02 -1.622118E-82 -2.005817E-80 -1.387778E-83 -1.713249E-81 ...
1.500000E-02 -9.020043E-80 -1.114231E-77 -4.402893E-81 -5.424313E-79 ...
2.000000E-02 -1.201684E-77 -1.467704E-75 -4.549845E-79 -5.533119E-77 ...
2.500000E-02 -1.528953E-75 -1.866265E-73 -4.126064E-77 -5.004886E-75 ...
# ... and so on.
```

Part 10: Contour-Plot File Format

There is one contour-plot file, as shown here:

Contour-plot file for TPV26 and TPV27	
File Name	Description
cplot	Rupture times for the fault.

The contour plot file lists the locations of all the nodes on the fault surface, and the time at which each node ruptures.

The contour plot file is an ASCII file that contains three data fields, as follows.

Contour Plot Data Fields for TPV26 and TPV27	
Field Name	Description, Units, and Sign Convention
j	Distance along strike (m). Sign convention: Positive means a location to the right of the origin. For TPV26 and TPV27, the value of <i>j</i> can range from -20000 to 20000.
k	Distance down-dip (m). Sign convention: Zero is the earth's surface, and positive means underground . For TPV26 and TPV27, the value of <i>k</i> can range from 0 to 20000.
t	Rupture time (s). This is the time at which fault slip-rate first changes from 0 to greater than 0.001 m/s. If this node never ruptures, use the value 1.0E+09.

A pair of numbers (*j*, *k*) denotes a point on the fault surface. It is equal to the (*x*, *y*) coordinates.

The contour plot file consists of three sections, as follows.

Contour Plot File Format for TPV26 and TPV27	
File Section	Description
File Header	<p>A series of lines, each beginning with a # symbol, that gives the following information:</p> <ul style="list-style-type: none"> • Benchmark problem (TPV26 or TPV27) • Author • Date • Code • Code version • Node spacing or element size • Descriptions of data columns (7 lines) • Anything else you think is relevant
Field List	<p>A single line, which lists the names of the 3 data fields, in column order, separated by spaces. It should be:</p> <p style="text-align: center;">j k t</p> <p>(all on one line). The server examines this line to check that your file contains the correct data fields.</p>
Rupture History	<p>A series of lines. Each line contains three numbers, which give the (j, k) coordinates of a node on the fault surface, and the time t at which that node ruptures.</p> <p>C/C++ users: We recommend using 14.6E or 14.6e floating-point format.</p> <p>Fortran users: We recommend using E15.7 or 1PE15.6 floating-point format.</p> <p>If a node never ruptures, the time should be given as 1.0E+09.</p> <p>Nodes may be listed in any order.</p>

Note: The nodes may appear in any order. The nodes do not have to form a rectangular grid, or any other regular pattern.

Note: When you upload a file, the server constructs the Delaunay triangulation of your nodes. Then, it uses the Delaunay triangulation to interpolate the rupture times over the entire fault surface. Finally, it uses the interpolated rupture times to draw a series of contour curves at intervals of 0.5 seconds.

Here is an example of a contour-plot file. This is an invented file, not real modeling data.

```
# Example contour-plot file.
#
# This is the file header:
# problem=TPV26
# author=A.Modeler
# date=2014/01/23
# code=MyCode
# code_version=3.7
# element_size=100 m
# Column #1 = horizontal coordinate, distance along strike (m)
# Column #2 = vertical coordinate, distance down-dip (m)
# Column #3 = rupture time (s)
#
# The line below lists the names of the data fields.
# It indicates that the first column contains the horizontal
# coordinate (j), the second column contains the vertical
# coordinate (k), and the third column contains the time (t).
j k t
#
# Here is the rupture history
-6.000000E+02 7.000000E+03 3.100000E-02
-6.000000E+02 7.100000E+03 4.900000E-02
-6.000000E+02 7.200000E+03 6.700000E-02
-7.000000E+02 7.000000E+03 1.230000E-01
-7.000000E+02 7.100000E+03 1.350000E-01
-7.000000E+02 7.200000E+03 1.470000E-01
# ... and so on.
```