EQdyna

An Explicit Finite Element Method (FEM) Code
for Modeling Spontaneous Dynamic Rupture On Geometrically Complex Faults

Benchun Duan
(Department of Geology & Geophysics, Texas A&M University, College Station, TX 77843-3115)

1. Overview of the Code

- Code name: EQdyna
- Current version: 2.0
- Type of code: Finite Element Method (FEM)
- Name of developer: Benchun Duan
- Special features used for spontaneous rupture problems:
  - Stiffness hourglass control
  - Stiffness-proportional Rayleigh damping (artificial viscous damping scheme).
- Code availability: the executable code is available from the developer upon request.
- Funding sources for code development and related work: NSF, SCEC, USGS

2. Technical Description (modified from Duan and Oglesby, 2006)

After discretizing the space domain into nonoverlapping elements, the standard FEM formulation for an elastodynamic problem with viscous damping [e.g., Hughes, 2000] leads to a semidiscrete (time is left continuous) matrix equation

$$\mathbf{Ma} + \mathbf{Cv} + \mathbf{Ku} = \mathbf{F}\,, \qquad (1)$$

where M is the mass matrix, C is the viscous damping matrix, K is the stiffness matrix, F is the vector of applied forces, and u, v, a are the displacement, velocity and acceleration vectors, respectively. A convenient form of C is the Rayleigh damping matrix

$$\boldsymbol{C} = \boldsymbol{pM} + \boldsymbol{qK}\,, \qquad (2)$$

where p and q are numerical parameters. The two components of Rayleigh damping are mass and stiffness proportional, with the former dominant at low frequency and the latter dominant at high frequency. To suppress high frequency numerical noises with the least effect on low frequency signals in models, one can set the mass proportional parameter p equal zero and only keep the stiffness proportional parameter q in (2). Then equation (1) can be written as

$$\boldsymbol{Ma} + \boldsymbol{K}(\boldsymbol{u} + \boldsymbol{qv}) = \boldsymbol{F}\,. \qquad (3)$$

The initial value problem for (1) or (3) needs two initial conditions

$$u(0) = u_{0}\,, \qquad (4a)$$

$$v(0) = v_{0}\,. \qquad (4b)$$

One of the most widely used methods for solving (3) to (4) is the central difference time integration method, which is explicit when the mass matrix M is diagonal:

$$\boldsymbol{a_n} = \boldsymbol{M^{-1}}\big(\boldsymbol{F_n} - \boldsymbol{K}(\boldsymbol{u_n} + \boldsymbol{qv_n})\big) \qquad (5a)$$

$$\boldsymbol{v_{n+1}} = \boldsymbol{v_n} + \boldsymbol{a_n}\,\Delta\boldsymbol{t}\,, \qquad (5b)$$

$$\boldsymbol{u_{n+1}} = \boldsymbol{u_n} + \boldsymbol{v_{n+1}}\,\Delta\boldsymbol{t}\,, \qquad (5c)$$

where n and n + 1 denote two consecutive time steps.

The diagonal mass matrix M can be obtained through lumping techniques. We use the technique that was proposed by Hinton et al. [1976]. The idea of this technique is to set the entries of the lumped mass matrix proportional to the diagonal entries of the consistent mass matrix (the mass matrix obtained from standard FEM formulation). The advantage of an explicit FEM may be seen from (5a) when M is diagonal. In this case, the solution may be advanced without the necessity of solving a coupled set of equations. The central difference method is conditionally stable, and the characteristic time step to ensure stability is determined by the minimum element size and wave speed in the model.

For computational efficiency, we employ quadrilateral elements in two dimensions and hexahedral elements in three dimensions with one-point integration. Previous studies suggest that the rate of convergence of the one-point quadrature element is comparable to that of fully integrated elements (four integration points in two dimensions and eight integration points in three dimensions) [Belytschko et al., 1984]. The major drawback of one-point quadrature in these elements is the existence of hourglass modes, which lead to hourglass instability in dynamic codes. We adopt the method proposed by Kosloff and Frazier [1978] to implement the hourglass control in our code EQdyna. The basic idea is to determine the element restoring forces to resist hourglass modes. This hourglass resistance H is added onto the right-hand side of equation (3), leading to a modified solution of equation (5a) as

$$\boldsymbol{a_n} = \boldsymbol{M^{-1}}(\boldsymbol{F_n} - \boldsymbol{K}(\boldsymbol{u_n} + \boldsymbol{q v_n}) + \boldsymbol{H_n}). \quad \textbf{(6)}$$

The crucial feature of the dynamic FEM for modeling spontaneous earthquake rupture is the implementation of the fault boundary condition. We use the traction-at-split node (TSN) method, which has been widely used, to characterize faults in our models. In the old version EQdyna 1.0, we implemented the TSN by following Andrews [1999]. In the current version EQdyna 2.0, we adopt the formation of the TSN given by Day et al. (2005), which provides a consistent treatment for fault behavior (at a given pair of split nodes) at all times, including prerupture, initial rupture, arrest of sliding, and possible reactivation and arrest of sliding. A slip-weakening friction law [Ida, 1972; Palmer and Rice, 1973; Andrews, 1976; Day, 1982] is implemented in the current version of the code EQdyna. The traction on faults only affects solutions of split nodes along faults, with the coupling force R added to one side of the fault and subtracted from the other side. Then for these split nodes, the solution of equation (6) is modified as

$$\boldsymbol{a_n} = \boldsymbol{M^{-1}}(\boldsymbol{F_n} - \boldsymbol{K}(\boldsymbol{u_n} + \boldsymbol{q v_n}) + \boldsymbol{H_n} \pm \boldsymbol{R_n}). \quad (7)$$

The slip-weakening friction law in this code can be expressed as

$$\boldsymbol{f}(\Delta \boldsymbol{u}) = \boldsymbol{f_d} + (\boldsymbol{f_s} - \boldsymbol{f_d})\left(\boldsymbol{1} - \frac{\Delta \boldsymbol{u}}{\boldsymbol{D_0}}\right)\boldsymbol{H}\left(\boldsymbol{1} - \frac{\Delta \boldsymbol{u}}{\boldsymbol{D_0}}\right), \quad (8)$$

where fs and fd are the static and dynamic coefficients of friction, respectively. D_0 is slip on the fault, H( ) is the Heaviside function, and D0 is the critical slip distance.

As a finite element code, EQdyna is designed to handle complex fault geometry. It can deal with different types of elements, such as quadrilateral elements in 2D, and hexahedral and tetrahedral elements in 3D by the degeneration technique (e.g., Hughes, 2000).

In choosing parameter q in above equations, we follow the suggestions given by Day et al. (2005) and Dalguer and Day (2007). In their finite difference method code, they have shown that the damping parameter q in equation (3) can be chosen to be proportional to the dynamic simulation time step Δt, i.e., q = βΔt. Δt is usually proportional to the minimum element size Δx in the model to ensure numerical stability, i.e., Δt = αΔx/Vp, where Vp is the P wave velocity and α is a constant (i.e., the Courant-Friedrich-Lwey (CFL) number, between 0 and 1). Thus, the damping parameter q can be proportional to the minimum element size Δx in the model, i.e.,

$$q = \beta\alpha\Delta x / v_p \ . \qquad (9)$$

3. An example of applying the code to SCEC benchmark problems: SCEC TPV6

Finite element mesh: within the main region of 30 km x 15 km x 1 km (the fault is in the center along the third direction), cubic block elements with a side length of 100 m are used. Surrounding the main region is a buffer region to avoid artificial boundary reflections from contaminating fault rupture propagation and recordings at selected stations. Within the buffer region, the side length of the adjacent elements increases by a ratio of 1.08 starting from the edges of the main region. The outer boundaries of the entire model region are fixed boundaries. The total numbers of elements and nodes in the model are 4,194,000 and 4,324,863, respectively.

Simulation time step is calculated by Δt = αΔx/Vp with α = 0.4, Δx = 100 m and Vp = 6000 m/s (the faster Vp in the model), which gives Δt = 0.0067 s. Output time interval is set to be 0.06 s (about once every 9 simulation time steps).

Stiffness-proportional **viscous damping parameters** (see equation 9): β is 0.1 for the main region and 0.3 for the buffer region, with the time step given above.

This problem was run on SDSU's old server "altai" with 1 CPU in February of 2007. It took about 27 hours and less than 4 GB RAM to run 12 s rupture propagation on the machine.

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