SEAS Benchmark Problem BP1-FD

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October 23, 2019

The problem set-up for benchmark BP1-FD is identical to that of the first benchmark problem (BP1/BP1-QD) except for the use of full elastodynamics instead of radiation damping to approximate the inertia effects. The total simulation time is reduced, but all other parameters are unchanged. Locations for time series output are same as BP1.

1   2D Problem Setup

The medium is assumed to be a homogeneous, isotropic, linear elastic half-space defined by

$$(x, y, z) \in (-\infty, \ \infty) \times (-\infty, \ \infty) \times (0, \ \infty),$$

with a free surface at $z = 0$ and z as positive downward. A vertical, strike-slip fault is embedded at $x = 0$, see Figure 1. We assume antiplane shear motion, letting $u = u(x, z, t)$ denote the displacement in the y-direction. Motion is governed by the momentum balance equation

$$\rho \frac{\partial^2 u}{\partial t^2} = \frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yz}}{\partial z}, \tag{1}$$

in the domain $(x, z) \in (-\infty, \infty) \times (0, \infty)$, where ρ is the material density. Hooke's law relates stresses to strains by

$$\sigma_{xy} = \mu \frac{\partial u}{\partial x}; \quad \sigma_{yz} = \mu \frac{\partial u}{\partial z} \tag{2}$$

for shear modulus μ.

2   Boundary and Interface Conditions

We supplement equations (1)–(2) with one boundary condition and two interface conditions. A free surface lies at $z = 0$, where all components of the traction vector equal 0. Mathematically, this condition is given by

$$\sigma_{yz}(x, 0, t) = 0. \tag{3}$$

At $x = 0$, the fault defines the interface. Superscripts "+" and "−" refer to the side of the fault with x positive, and x negative, respectively. We define slip by $\delta(z, t) = u(0^+, z, t) - u(0^-, z, t)$, i.e. the jump in displacement across the fault, with right-lateral motion yielding positive values of δ. We require that components of the traction vector be equal and opposite, which reduces in antiplane shear to the condition

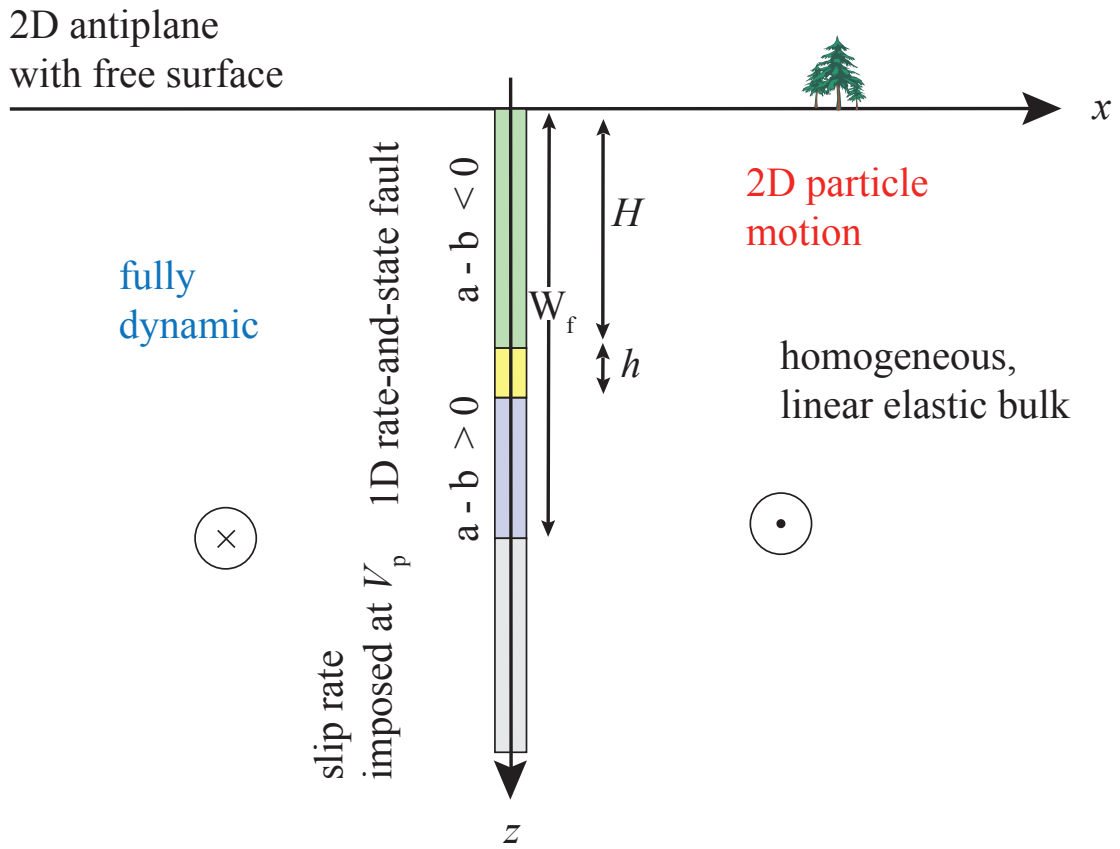$$\sigma_{xy}(0^+, z, t) = \sigma_{xy}(0^-, z, t), \tag{4}$$

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Figure 1: This initial benchmark considers a planar fault embedded in a homogeneous, linear elastic half-space with a free surface. The fault is governed by rate-and-state friction down to the depth W_{f} and creeps at an imposed constant rate V_{p} down to the infinite depth. The simulations will include the nucleation, propagation, and arrest of earthquakes, and aseismic slip in the post- and inter-seismic periods.

and denote the common value by $\Delta\tau$ (shear stress perturbation due to antiplane deformation). Note that positive values of $\Delta\tau$ denotes stress that tends to cause right-lateral motion.

The second interface condition is depth dependent. Down to a depth of W_{f}, we impose rate-and-state friction, namely, that shear stress on the fault be equal to fault strength F, namely

$$\tau = F(V, \theta), \tag{5}$$

where $\tau = \tau^0 + \Delta\tau$ is the sum of the prestress, and the shear stress perturbation. The fault strength $F = \sigma_{\mathrm{n}} f(V, \theta)$, where $V = \frac{\partial u}{\partial t}(0^+, z, t) - \frac{\partial u}{\partial t}(0^-, z, t)$ is the slip rate, and θ is the state variable. σ_{n} is the effective normal stress on the fault. For this first benchmark problem we assume σ_{n} is constant, given in Table 1. θ evolves according to the aging law

$$\frac{d\theta}{dt} = 1 - \frac{V\theta}{L}, \tag{6}$$

where L (denoted D_{c} is previous benchmarks) is the critical slip distance. The friction coefficient f is given by a regularized formulation

$$f(V, \theta) = a \sinh^{-1}\left[\frac{V}{2V_0} \exp\left(\frac{f_0 + b\ln(V_0\theta/L)}{a}\right)\right] \tag{7}$$

for reference friction coefficient f_0, reference slip rate V_0, and rate-and-state parameters a and b. For this benchmark, b is constant as b_0 and a varies with depth (insert in Figure 1) as follows:

$$a(z) = \begin{cases} a_0, & 0 \le z < H \\ a_0 + (a_{\max} - a_0)(z - H)/h, & H \le z < H + h \\ a_{\max}, & H + h \le z < W_{\mathrm{f}} \end{cases} \tag{8}$$

Below depth W_{f}, the fault creeps at an imposed constant rate, given by the interface condition

$$V(z, t) = V_{\mathrm{p}}, \quad z \ge W_{\mathrm{f}}, \tag{9}$$

where V_{p} is the plate rate.

3   Initial Conditions and Simulation Time

Initial conditions on particle displacement and velocity are required, as well as on the state variable. We assume that displacements are initially zero everywhere in the domain, i.e.

$$u(x, z, 0) = 0. \tag{10}$$

Particle velocities are given by

$$v(x^+, z, t) = V_{\mathrm{init}}/2 \tag{11}$$

$$v(x^-, z, t) = -V_{\mathrm{init}}/2 \tag{12}$$

where V_{init} is specified.

Table 1: Parameter values used in first benchmark problem

Parameter	Definition	Value, Units
ρ	density	2670 kg/m^3
c_s	shear wave speed	3.464 km/s
σ_n	effective normal stress on fault	50 MPa
a_0	rate-and-state parameter	0.010
a_max	rate-and-state parameter	0.025
b_0	rate-and-state parameter	0.015
L	critical slip distance	0.008 m
V_p	plate rate	10^{-9} m/s
V_init	initial slip rate	10^{-9} m/s
V_0	reference slip rate	10^{-6} m/s
f_0	reference friction coefficient	0.6
H	depth extent of uniform VW region	15 km
h	width of VW-VS transition zone	3 km
W_f	width of rate-and-state fault	40 km
Δz	suggested cell size	25 m
t_f	final simulation time	1500 years

The initial state on the fault is chosen to be consistent with a uniform slip rate and pre-stress at constant values V_init and τ^0, respectively. The pre-stress τ^0 corresponds to the steady-state stress with slip rate V_init at the depth of W_f, namely

$$\tau^0 = \sigma_\text{n} a_\text{max} \sinh^{-1} \left[\frac{V_\text{init}}{2V_0} \exp \left(\frac{f_0 + b_0 \ln(V_0/V_\text{init})}{a_\text{max}} \right) \right]. \tag{13}$$

To be consistent with slip rate and pre-stress everywhere, the initial state is variable with depth and not necessarily at the steady state:

$$\theta(z, 0) = \frac{L}{V_0} \exp \left\{ \frac{a}{b} \ln \left[\frac{2V_0}{V_\text{init}} \sinh \left(\frac{\tau^0}{a\sigma_\text{n}} \right) \right] - \frac{f_0}{b} \right\}. \tag{14}$$

Equations (1)–(2), along with boundary condition (3), interface conditions (4)–(5) and (9), and initial conditions (10), (11) and (14) are solved over the time period $0 \le t \le t_\text{f}$, where t_f is a specified final simulation time. All necessary parameter values for this benchmark problem are given in Table 1.

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4 On-fault Time Series Output

Files are uploaded to the SCEC code validation web server at this address:

    http://scecdata.usc.edu/cvws/cgi-bin/seas.cgi

You need to upload on-fault ($x = 0$) time series files, which give slip δ, base 10 log of the slip rate V, base 10 log of the state variable (i.e. $\log_{10}(\theta)$), and shear stress τ, for each on-fault station at representative time steps. We define the simulation periods as either aseismic (when $\max(V) < 10^{-3}$ m/s, where $\max(V)$ is the maximum slip rate over the entire fault) or seismic (when $\max(V) \geq 10^{-3}$ m/s). When outputting modeling results, use larger time intervals (e.g., $\sim$0.1 yr) during aseismic periods and smaller time intervals (e.g., $\sim$0.1 s) during seismic periods. More variable time steps are OK. Please keep the total number of time steps in the data file on the order of 10^4–10^5.

Time series data is supplied as ASCII files, one file for each station. There are 12 stations in total, as follows:

    `fltst_dp000`: $z = 0$ km (at the free surface)
    `fltst_dp025`: $z = 2.5$ km
    `fltst_dp050`: $z = 5$ km
    `fltst_dp075`: $z = 7.5$ km
    `fltst_dp100`: $z = 10$ km
    `fltst_dp125`: $z = 12.5$ km
    `fltst_dp150`: $z = 15$ km
    `fltst_dp175`: $z = 17.5$ km
    `fltst_dp200`: $z = 20$ km
    `fltst_dp250`: $z = 25$ km
    `fltst_dp300`: $z = 30$ km
    `fltst_dp350`: $z = 35$ km

Each time series has 5 data fields, as follows.

Field Name	Description, Units and Sign Convention
`t`	Time (s)
`slip`	Out-of-plane slip (m). Positive for right-lateral motion.
`slip_rate`	$\log_{10}$ of the out-of-plane slip-rate ($\log_{10}$ m/s). Positive for right-lateral motion.
`shear_stress`	Shear stress (MPa). Positive for shear stress that tends to cause right-lateral motion.
`state`	$\log_{10}$ of state variable ($\log_{10}$ s).

The on-fault time series file consists of three sections, as follows:

File Section	Description
File Header	A series of lines, each beginning with a # symbol, that give the following information: • Benchmark problem (No.3) • Code name • Code version (optional) • Modeler • Date • Node spacing or element size • Station location • Minimum time step (optional) • Maximum time step (optional) • Number of time steps in file (optional) • Anything else you think is relevant (optional) • Descriptions of data columns (5 lines)
Field List	A single line, which lists the names of the 5 data fields, in column order, separated by spaces. It should be: `t slip slip_rate shear_stress state` (all on one line). The server examines this line to check that your file contains the correct data fields.
Time History	A series of lines. Each line contains 5 numbers, which give the data values for a single time step. The lines must appear in order of increasing time. **Make sure to use double-precision when saving all fields**. **C/C++ users:** We recommend using 21.13E or 21.13e floating-point format for the time field and 14.6E or 14.6e format for all other data fields. **Fortran users:** We recommend using E22.14 or 1PE22.13 floating-point format for the time field and E15.7 or 1PE15.6 format for other data fields. The server accepts most common numeric formats. If the server cannot understand your file, you will see an error message when you attempt to upload the file.

Here is an example of an on-fault time-series file, with invented data.

```
# This is the file header:
# problem=SEAS Benchmark No.3
# code=MYcode
# version=1.0
# modeler=A.Modeler
# date=2018/10/01
# element_size=35 m
# location= on fault, 7.2km depth
# minimum_time_step=0.1
# maximum_time_step=3.157e6
# num_time_steps=2400
# Column #1 = Time (s)
# Column #2 = Slip (m)
# Column #3 = Slip rate (log10 m/s)
# Column #4 = Shear stress (MPa)
# Column #5 = State (log10 s)
#
```

```
# The line below lists the names of the data fields
t slip slip_rate shear_stress state
# Here is the time-series data.
0.000000E+00 0.000000E+00 -9.000000E+00 3.000000E+01 7.000000E+00 ...
5.000000E-02 0.000000E+00 -9.000000E+00 3.000000E+01 7.000000E+00 ...
1.000000E-01 0.000000E+00 -9.000000E+00 3.000000E+01 7.000000E+00 ...
1.500000E-01 0.000000E+00 -9.000000E+00 3.000000E+01 7.000000E+00 ...
# ...  and so on.
```

5   Slip and Stress Evolution Output

The slip and stress evolution output files, with the names

`slip.dat` and `stress.dat`

are two ASCII files that record the spatial distribution of slip and shear stress τ, respectively, on a subset of fault nodes at representative time steps during the aseismic and seismic phases of the simulation. Data can be saved using representative time intervals of approximately 5 yr and 1 s for results in aseismic and seismic phases, respectively, or with variable time steps. Data will be interpolated to plot slip every 5 yr during the aseismic phase, and every 1 s during the coseismic phase. The data should include a subset of nodes (e.g., with a spacing of $\sim$500 m) down to a depth of 40 km (the width of rate-and-state frictional fault, W_{f}). The file also contains the time series of maximum slip rates (taken over the entire fault), so that we can precisely differentiate aseismic and seismic phases.

NOTE: Please upload this data to a Dropbox folder that will be shared with you (or send request to bae@uoregon.edu). Please ensure that the file is $\sim$10s of MBs or less.

The data file has 4 data fields, as follows:

Field Name	Description, Units and Sign Convention
z	Depth (m) at approximately 500 m increments, down to 40 km. Positive for all depths.
t	Time (s). Nonuniform time steps.
max_slip_rate	The $\log_{10}$ of maximum out-of-plane slip-rate (taken over the entire fault) ($\log_{10}$ m/s). Positive for right-lateral motion.
slip OR stress	Out-of-plane slip (m) OR stress (MPa). Positive for right-lateral motion.

The data output consists of three sections, as follows:

File Section	Description
File Header	A series of lines, each beginning with a # symbol, that give the following information: • Benchmark problem (No.3) • Modeler • Date • Code • Code version (if desired) • Node spacing or element size • Descriptions of data fields (3 lines) • Anything else you think is relevant (e.g. computational domain size)
Field List	Two lines. The first line lists z. The second lists the names of the 3 other data fields on one line, separated by spaces. It should be: `z` `t max_slip_rate slip` OR `stress` (last three fields on one line).
Slip OR Stress History	A series of lines that form a 2-dimensional array of rows and columns. The first row/line lists the numbers 0, 0 (to maintain a consistent array size), followed by the spatial nodes with increasing depth as you go across the row. Starting from the second row/line, each row/line contains time, maximum slip rate, and slip of all nodes at the time. These lines appear in order of increasing time (from top to bottom) and slip is recorded with increasing depth (from left to right). **Make sure to use double-precision when saving all fields.** **C/C++ users:** We recommend using 21.13E or 21.13e floating-point format for the time field and 14.6E or 14.6e format for all other data fields. **Fortran users:** We recommend using E22.14 or 1PE22.13 floating-point format for the time field and E15.7 or 1PE15.6 format for other data fields.

Note that z should appear in the first row, preceded by two zero numbers, for nodes with a spacing of approximately 500 m down to a depth of 40 km (width of rate-and-state frictional fault). Time and maximum slip rate should appear as two single columns that start on the second row, with time increasing as you go down. Slip/stress history (the remaining block) is represented by a two-dimensional array with time increasing as you go down the rows/lines, and z increasing as you go across the columns (approximately 81 columns). The entire output array should be of size $(N_t + 1, \sim83)$, where N_t is the total number of time steps. This means that you output slip/stress at selected nodes at one time step and move on to the next time step. (To keep the file on the order of 10s of MB, N_t should be on the order of 10,000).

The two-dimensional array should therefore be of the form: $\begin{bmatrix} 0 & 0 & z \\ T & \max(V) & \text{slip/stress} \end{bmatrix}$

Here is an example of a slip-evolution file for `slip.dat`, with invented data.

```
# This is the file header:
# problem=SEAS Benchmark No.3
# modeler=A.Modeler
# date=2019/03/01
# code=MyCode
```

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```
# code_version=3.7
# element_size=25 m
# Row #1 = Depth (m) with two zeros first
# Column #1 = Time (s)
# Column #2 = Max slip rate (log10 m/s)
# Columns #3-83 = Slip (m)
# Computational domain size:  depth 100 km, distance off fault 100 km
# The line below lists the names of the data fields
z
t max_slip_rate slip
# Here are the data
0.000000E+00   0.000000E+00 0.000000E+00 5.000000E+02 ...  4.000000E+04
0.000000E+00 -9.000000E+00 0.000000E+00 0.000000E+00 ...  0.000000E+00
3.140000E+05 -9.000000E+00 1.340000E-05 1.340000E-05 ...  3.140000E-05
1.227000E+07 -9.000000E+00 1.560000E-05 1.560000E-05 ...  1.220000E-02
4.690000E+07 -9.000000E+00 1.580000E-05 1.580000E-05 ...  4.680000E-02
...
9.467078E+10 -4.500000E-01 9.050000E+01 9.050000E+01 ...  9.461000E+01
```

6   Using the Web Server

The web server lets you upload your modeling results (section 4). Once uploaded, you and other modelers can view the data in various ways.

6.1   Logging in and Selecting a Problem

To log in, start your web browser and go to the home page at:

    http://scecdata.usc.edu/cvws/cgi-bin/seas.cgi

Click on "Upload Files," and then log in using your user name and password. Remember that passwords are case-sensitive. You are then presented with a list of code validation problems. Find the problem you want, and click the "Select" button. You will then see a list of files for the problem.

6.2   Navigating the Site

You navigate through the site by clicking buttons on the web pages. Avoid using your browser's Back or Forward buttons. If you use the Back or Forward buttons, you may get error messages from your browser.

6.3   Uploading Files

To upload a file, do the following steps:

- Find the file you want to upload, and click "Select." The server displays a page for you to upload the file.

- Select the data file on your computer. The exact method for file selection varies depending on operating system and web browser.

- Click on "Click Once to Upload." The file you selected is uploaded to the server.

When you upload a file, the web server immediately checks for correct formatting. There are three possible results:

- If the file is correctly formatted, the server displays a page noting the successful upload.

- If the file contains errors, the server displays an error log. The error log lists the errors that were detected in the file, each identified as specifically as possible.

- If the file is correctly formatted, but is questionable in some way (for example, a missing time step), then the server displays a warning log, which describes the problem.

When uploading time series files, the website may issue a warning that the time series cannot be filtered. Modelers should ignore this warning. After uploading a file, the file list shows the date and time that you uploaded the file. Remember that any file you upload will be visible to anyone who has access to the web site.

Additional help is available by clicking the "Help" link in the upper right corner of the webpage. Modelers who want to upload multiple versions of the benchmark (for example, using different element sizes), can do so using the "Change Version" feature of the website, which is described in the help screens. Direct further questions to Michael Barall.

6.4   Graphing, Viewing, and Deleting Files

After uploading a file, additional functions become available. These functions let you graph, view, or delete the uploaded file.

Graphing: To graph a file, find the file you want and click "Graph." For a time-series file, the server displays graphs of all the data fields in the file. At the bottom of each graph page, there is a box you can use to adjust graphing preferences. Graphing a file is a good way to check that the server is interpreting your data as you intended.

Viewing: To view the text of a file, find the file you want and click "View."

Deleting: To delete a file from the server, find the file you want and click "Delete." The server displays a page asking you to confirm the deletion.

7   Benchmark Tips

Numerical boundary conditions (to truncate the half-space when defining the computational domain) will most likely change results at least quantitatively, or even qualitatively. We suggest extending these boundaries until you see results appear independent of the computational domain size.

Other aspects that could influence results are the switching of solvers/time stepping schemes between seismic/aseismic phases (e.g. the technique for choosing when to switch between quasi-static and dynamic codes depending on the regime and/or differences in boundary conditions for the different regimes). We suggest exploring these criteria to find robust results that do not differ (at least qualitatively) on when this switch occurs. We prefer participants to use the cell-size suggested in Table 1 and welcome results for different spatial resolutions.